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The View Across the Silos®

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Between the ISSUES

The Retirement Management Journal, a publication of the Retirement Income Industry Association, is devoted exclusively to retirement-income planning and management.

Monitoring and Managing a Retirement Income Portfolio

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Preface by Francois Gadenne, Chairman of the Board and Executive Director of RIIA®

RIIA's mission is to discover, validate and teach the new realities of retirement from its unique perspective of the view across the business silos.

This mission makes RIIA the most objective and comprehensive map-maker of the entire retirement territory as presented in the RMA® Curriculum in general and the Procedural Prudence map shown below in particular.

This important paper explores key concepts that can be traced on the Procedural Prudence map as follows:

- "Feasibility" that belongs in the Client Diagnostic Kit/Shared Expectations block,
- "Sustainability" that belongs in the Client Diagnostic Kit/Key Policy Decisions block, and
- "Monitoring" that belongs in RMA Practice Management/Execution, Performance Measurement and Monitoring blocks.

RETIREMENT INCOME riia® INDUSTRY ASSOCIATION

RIIA®'s RMA® PROCEDURAL PRUDENCE MAP: A Summary of Wealth and Consumption Planning℠ Levels of Service

ADVISOR-PROVIDED DATA
rma CLIENT DIAGNOSTIC KIT — CH. 1 RPS #1

- ☐ Capital market expectations
- ☐ Household Balance Sheet (HHBS) Benchmarks℠

CLIENT-PROVIDED DATA
rma CLIENT DIAGNOSTIC KIT — CH. 1 RPS #2

- ☐ Funded ratio analysis with the measures of fundedness (overfunded, constrained, underfunded)
- ☐ Collection of Budget/Income & Expense Statement data
- ☐ Collection of the Household Balance Sheet data (human, social, financial, and physical capital, as well as liabilities)
- ☐ Mapping of systematic, portfolio view risk exposures (Sequence of Return Risk Profile)
- ☐ Mapping of unsystematic, client view risk exposures (Sequence of Consumption Risk Profile)

KEY POLICY DECISIONS
rma CLIENT DIAGNOSTIC KIT — CH.'S 2 & 3 RPS #3, 4.a

- ☐ Budget/Income & Expense Statement projections over the retirement lifecycle horizon
- ☐ Household Balance Sheet projections over the retirement lifecycle horizon

Based on Measures of Fundedness:
- ☐ Fundamental planning decision
  - ☐ Investment-based planning
  - ☐ Aspirational-based planning (Existential Attitudes/Learning Identities/HC Abilities/Client Typologies)
  - ☐ Goals-based planning
  - ☐ Product-based planning
- ☐ Data-gathering approach
  - ☐ Incremental vs. Comprehensive

Based on the Household Balance Sheet:
- ☐ Capital management style
  - ☐ Conservative vs. Dissipative
- ☐ Portfolio implementation style
  - ☐ One vs. Multiple
- ☐ Flooring determination methodology
  - ☐ Formula/HHBS/Qualitative

Based on the Budget/Income & Expense Statement:
- ☐ Flooring duration/Horizon
- ☐ Front-loaded/Back-loaded/Smoothed/Variable standard of living cash-flow pattern
- ☐ Flooring expense
  - ☐ Budget vs. Insure
- ☐ Monitoring frequency

Based on the Risk Exposure Map:
- ☐ Solidity of flooring
  - ☐ Probability-based/Safety-first/Guarantees
- ☐ Risk-monitoring thresholds
  - ☐ Levels of impairment

SHARED EXPECTATIONS
rma CLIENT DIAGNOSTIC KIT — CH.'S 2 & 3 RPS #4.b, 5

ASSETS

PV of Human Capital (HC):
- ☐ Employer Contributions
- ☐ W2/1099 Income

PV of Social Capital (SC):
- ☐ Social Security analysis
- ☐ Pension decision analysis
- ☐ Income from trust analysis
- ☐ Expected inheritance analysis

PV of Financial Capital (FC):
- ☐ Tax-advantaged accounts
- ☐ Tax-free accounts
- ☐ Taxable accounts

Value of Other Assets:
- ☐ Primary home
- ☐ Other real estate
- ☐ Business ownership
- ☐ Vehicles (planes, boats, autos, etc.)
- ☐ Tangible property

Contractual Protection:
- ☐ Life insurance
- ☐ Disability insurance
- ☐ Long-term care insurance
- ☐ P&C insurance
- ☐ Liability/Umbrella insurance
- ☐ Professional insurance

LIABILITIES

PV of Expected Consumption in Retirement:
- ☐ Fixed/Non-discretionary/Essential
- ☐ Rising/Non-discretionary/Essential
- ☐ Healthcare/Medicare decision analysis
- ☐ Variable/Discretionary/Non-essential

Debt:
- ☐ Primary mortgage
- ☐ Primary home equity loan(s)
- ☐ Reverse mortgage
- ☐ Other real-estate loans
- ☐ Education loans
- ☐ Vehicle loans (planes, boats, autos, etc.)
- ☐ Credit card debt
- ☐ Other debt

Household Net-Worth (Risk Capacity Analysis)

Special Circumstances Checklist:
- ☐ Last Will & Testament and/or Revocable Trust
- ☐ Living Will
- ☐ Estate executor
- ☐ Durable Power of Attorney
- ☐ Married/Cohabit/LGBT
- ☐ Single/Divorced/Widowed
- ☐ Dependent children
- ☐ Dependent adults
- ☐ Special needs children/adults
- ☐ Other special circumstances

RETIREMENT ALLOCATIONS
rma RETIREMENT ALLOCATIONS — CH. 4 RPS #6, 7

Strategic Choices of Risk-Management Techniques Allocations:
- ☐ Retention/Diversification/Risky Assets - Upside Portfolio (%)
- ☐ Pooling/Insure vs. Budget/Insurance - Longevity Portfolio (%)
- ☐ Management/Prevention/Hedges - Floor Portfolio (%)
- ☐ Avoidance/Cushion/Cash - Reserves Portfolio (%)

PROCESS APPROACH(ES) IMPLEMENTATION
rma RMA TOOLBOX — CH. 5 RPS #8

Tactical Selection of Implementation Process Approaches:
- ☐ Engineering process approaches
  - ☐ Total return portfolios
  - ☐ Systematic withdrawal plans
    - ☐ Decision rules/market view/fixed
    - ☐ Actuarial/client view/variable
  - ☐ Floating floors
  - ☐ Time segmentation
  - ☐ Ladders
  - ☐ Rolling Floors
  - ☐ Guarantees
- ☐ Economic process approaches
  - ☐ Smoothing (Kotlikoff)
  - ☐ MRT (Branning)
  - ☐ Floor/Upside (Zwecher)
  - ☐ Safety-First (Bodie)
  - ☐ Barbelling (Taleb)

PORTFOLIO CONSTRUCTION
rma RMA TOOLBOX — CH. 6 RPS #9

- ☐ Account location choices (tax-free, tax-deferred, taxable)
- ☐ Product selection
  - ☐ Products that "wrap" the market
  - ☐ Products that "wrap" ladders
  - ☐ Products that "wrap" mortality
    - ☐ Long /life insurance
    - ☐ Short/annuities
  - ☐ Products that "wrap" an actuarial bond
  - ☐ Products that "wrap" structures across risk-management techniques
  - ☐ Products that "wrap" options
  - ☐ Products that "wrap" the HHBS
  - ☐ Products that "wrap" the price of income
  - ☐ Products that "wrap" debt
  - ☐ Products that "wrap" inflation
  - ☐ Products that "wrap" taxes
  - ☐ Products that "wrap" liquidity

EXECUTION
rma RMA PRACTICE MANAGEMENT — CH. 7 RPS #10

Written Retirement Policy Statement:
- ☐ Summary of Client Diagnostic Kit/Retirement Allocations/RMA Toolbox choices
- ☐ Retirement Policy Statement process checklist
- ☐ Planned implementation steps

Account Consolidation
- ☐ Bank Accounts
- ☐ Brokerage Accounts
- ☐ Asset Management Accounts
- ☐ Retirement Accounts

Buy/Sell list:
- ☐ Custom solutions
- ☐ Turnkey solutions
  - ☐ R-MAP (Lonier)
  - ☐ Asset Dedication (Burns)
  - ☐ Income-4-life (Macchia)

PERFORMANCE MEASUREMENT
rma RMA PRACTICE MANAGEMENT

MONITORING — rma RMA PRACTICE MANAGEMENT

MONITORING — rma RMA PRACTICE MANAGEMENT

MONITORING — rma RMA PRACTICE MANAGEMENT

MONITORING — rma RMA PRACTICE MANAGEMENT

Note: Chapter #'s refer to the chapters in the 2013 RMA Curriculum Book (5th Edition).
Note: RPS #'s refer to the RMA's Retirement Policy Statement steps identified in the 2014 RMA Update Bulletin #1, as well as a defined term in the Glossary.

THE VIEW ACROSS THE SILOS

Monitoring and Managing a Retirement Income Portfolio

PART ONE

Copyright: Patrick J. Collins, PhD, CFA, Huy Lam, CFA, Josh Stampfli, MS EESOR

I. Introduction

Successfully managing a retirement income portfolio can be difficult, especially during periods of financial asset price declines. This two-part essay discusses portfolio monitoring with respect to a number of risk metrics including ‘feasibility,’ and ‘solvency.’ These metrics differ from performance evaluation metrics—e.g., the Sharpe Ratio, the Information Ratio, and the Jensen’s Alpha measure; and from shortfall metrics—e.g., shortfall probability relative to periodic income or terminal wealth targets, mean expected shortfall, shortfall magnitude and, in the worst-case, portfolio depletion (bankruptcy) risk or, to use an alternate term, the ‘risk of ruin.’ Performance evaluation metrics help investors make inferences regarding *past* results. Shortfall risk metrics are *forward* looking and generate probabilities based either on (1) the assumption that history repeats, or (2) on projections derived from an investment risk model. The result is either a “groundhog’s day” view of investment risk or a reliance on outputs from models incorporating sometimes questionable assumptions regarding the nature ofinflation and asset price evolutions. By contrast, this essay outlines a portfolio monitoring system based, in large part, on *current* observables.

Some investors sidestep the issue of ongoing portfolio monitoring in favor of following pre-set, bright-line rules adopted at the time of portfolio implementation. In practice, such rules are often codified within a written Investment Policy Statement [IPS]. Under this static or 'architectural' view of Investment Policy, sticking to pre-set asset management and withdrawal rules offers a high probability of achieving a safe and sustainable lifetime income. Practitioners, however, have recently started to implement more dynamic 'systems engineering' approaches to Investment Policy Statements. We briefly discuss the new Investment Policy recommendations promulgated by the CFA Institute regarding the importance of establishing a policy anticipating that a portfolio might confront an unsustainable future drawdown. This shift in emphasis, perhaps arising from severe equity market downturns during the first decade of this century, augments the importance of effective portfolio surveillance and monitoring policy. Paradoxically, some commentators recommend responding to investment turbulence by replacing a single bright-line rule like the 'four-percent-withdrawal-rate' rule[1] with a veritable plethora of bright-line rules-for-all-occasions. By contrast, this essay does not parse historical return paths searching for an elusive oxymoron—namely, a set of rules-to-follow-for-a-random-process. Reliance on a single historical path of realized returns to develop and codify rules for portfolio control variables such as asset allocation and distribution policy is, at the limit, an elaborate exercise in data mining.[2]

Some practitioners base portfolio monitoring on outputs generated by portfolio risk models which often employ bootstrapping or Monte Carlo simulation techniques. Probability assessments reflect the number of economically successful trials divided by the total number of trials generated by the simulation application. However, model-based probability is not equivalent to "classical" probability calculations which rely on observation of empirical results such as rolls of a die or tosses of a coin. Rather, model-based probability relies on outputs generated by computer algorithms that approximate, with varying degrees of accuracy, the processes that drive financial asset price changes. Probability assessments are only as good as the models upon which they are based—that is to say, assessments are prone to 'model risk.' Thus, a portfolio monitoring and surveillance program should not over rely on outputs produced by risk models; and, any model used to monitor the portfolio should be academically defensible.[3]

Monitoring becomes especially important during times of poor investment performance—either due to adverse market conditions or to an investor's unsuccessful asset management elections. There are a number of responses to declining portfolio values. A common response is a 'sit tight' approach which advocates maintaining constant fixed-weight exposures to the portfolio's asset classes. The focus is on monitoring the current allocation's drift from the long-term strategic weights established in the IPS. This monitoring system is appropriate for risk models assuming investors manifesting Constant Relative Risk Aversion [CRRA]. CRRA assumes: (1) risk tolerance is invariant to changes in wealth, and (2) absent dramatic changes in investor health, longevity expectations, etc., the investor maintains a constant time preference rate for consumption.[4] Under this portfolio management

1 Each year, withdraw four percent of a portfolio's initial value with an adjustment for annual inflation. The result is a constant-dollar fixed retirement income stream. Commentators sometimes use the term 'synthetic annuity' to describe a self-managed fixed retirement income stream, and to distinguish it from an annuity contract underwritten by an insurance company. The nomenclature can, at times, become confusing. In this paper, unless otherwise indicated, an annuity refers to a contract obligating an insurer to provide periodic income to a designated annuitant.

2 One type of data mining occurs when (1) a time-series analysis uncovers patterns or parameters that best fit the sample data; and, (2) the same data is then used to develop and test the efficacy of asset management rules based on the patterns or parameters.

3 For an in depth discussion of retirement income risk modelling see, Collins, Patrick J. and Lam, Huy D. and Stampfli, Josh, How Risky is Your Retirement Income Risk Model? (May 13, 2015). Financial Services Review, Forthcoming. Available at SSRN: http://ssrn.com/abstract=2548651 or http://dx.doi.org/10.2139/ssrn.2548651

4 The investor's elasticity of intertemporal substitution is one ÷ risk aversion coefficient under a CRRA utility function. Some life-cycle models indicate that the optimal consumption path for individuals with CRRA preferences increases over time because future consumption is worth less than current consumption—i.e., to keep the utility of consumption constant, future consumption must increase. The rate of increase is a function of the force of mortality, and the level of consumption strongly depends on expectations for remaining lifespan. Other life-cycle models suggest that a high discount rate for future consumption leads to a preference for front-loaded retirement consumption. Investors with such a preference exhibit "Fisher utility" which leads to decreases in retirement spending over the planning horizon. In a model where the investor lacks any bequest motives, all remaining wealth is consumed in the period just prior to death. When the investor's time preference rate is exactly equal to the risk free rate, optimal

approach, a retiree suffering the effects of a bear market hopes for a market recovery sufficient to allow the portfolio to continue future target distributions. Many financial advisors caution clients to "trust the market," "stay the course," "maintain investment discipline," "avoid market timing," and so forth.

By contrast, dynamic asset management encompasses a variety of asset allocation / withdrawal strategies. Dynamic programming, for example, focuses on finding the optimal decision path given current resources—e.g., jointly considering asset allocation and consumption strategies by solving for formulae such as the Merton Optimum in a continuous time finance model. That is to say, investment and consumptions decisions are not separable. Alternately, a Portfolio Insurance strategy makes discrete time changes in asset weightings in response to changing portfolio values in order to replicate an option-like convex payoff structure. Finally, at the other end of the asset management spectrum, are found a variety of empirical rules [adaptive portfolio withdrawal strategies] purporting to yield maximum portfolio sustainability. These rules may lack a firm basis in economic theory, but are recommended to investors because they would have produced successful results under previous bear market conditions. However, it may be dangerous to apply retirement withdrawal rules that lack 'mathematical necessity,' and it is interesting to evaluate results when applying such rules to non-U.S. markets—e.g., to the Nikkei 225 stock index since its high water mark at the end of 1989.

Irrespective of the investor's initial portfolio management elections—'buy-and-hold,' 'constant-mix,' 'floor + multiplier,' 'tactical asset allocation,' 'bottom-up security selection,' 'top-down strategic asset allocation,' 'glide-path,' 'passive investment management,' 'active investment management,' 'benchmark relative,' 'asset/liability match,' etc.; and, irrespective of the initial elections for withdrawal management—'rules based,' 'fixed monthly amounts,' 'percentage of corpus amounts,' 'longevity relative,' etc., the critical objective is to assure that the portfolio can provide the required cash flows. Investors spend cash—not Information Ratios or Merton Optimums; and they need to know that the portfolio can sustain a suitable standard of living throughout their lifespan. The need to know whether the portfolio is in trouble is a primary justification for establishing an appropriate surveillance and monitoring program. Money management encompasses ongoing monitoring; and effective monitoring helps the investor assess the continued feasibility of retirement objectives relative to financial resources at hand. There exists a substantial body of academic research evaluating the merits of various combinations of the portfolio management / withdrawal strategies / asset allocation approaches listed above.[5] There is far less commentary on how to monitor the portfolio once it begins operations under the investment policy guidelines approved by the investor.

The need for effective portfolio monitoring increases during a bear market. If a portfolio suffers poor investment returns, it may be unable to fund an adequate and sustainable cash flow from a diminishing pool of capital. Although bear markets generally increase the desirability of capital preservation strategies, winding down equity risk in a low-interest rate environment may place the investor in a further bind. As stock values sink, a flight to quality often means acceptance of extraordinarily low yields on principal-guaranteed investments. Even at low rates of inflation, historically low yields on default-free investments cannot protect against erosion of purchasing power. The distributional flexibility provided by percentage-of-corpus withdrawal formulae may not be able to solve the dilemma of too little resources and too much cash flow demand. In a nutshell, the investor may find it difficult to remove fifteen pounds from a ten pound sack.

With that focus, the following questions arise: If a portfolio's market value falls, how does the investor determine if threshold retirement financial objectives continue to remain feasible? At what level of wealth does

initial retirement spending can be strikingly low. Only a highly risk averse investor would choose the extreme level of frugality in early retirement demanded by certain types of life-cycle models. An actuarial variation on such a spending trajectory is embedded in the US tax code's Minimum Required Distribution rules for IRAs.

5 For a review of the literature on the topic of 'Longevity and Portfolio Sustainability," see the approximately 400 page annotated bibliography available at: http://schultzcollins.com/static/uploads/2015/02/Annotated-Bibliography.pdf Additionally, the authors are currently writing a comprehensive literature review on this topic for the CFA Institute Research Foundation.

the investor determine that it is unlikely that the portfolio has a reasonable probability of successfully meeting critical objectives over the remaining planning horizon? How does the retired investor track a dynamically unfolding problem? We argue that a well-designed monitoring and surveillance program benefits investors by providing timely alerts regarding the scope of unfolding difficulties. It helps an investor distinguish between negative returns that are merely unpleasant and negative returns that are potentially catastrophic. Further, an effective monitoring system facilitates implementation of appropriate and timely adjustments. It enhances the ability to formulate prudent asset management responses so that investor reactions are not byproducts of excessive fear or greed.

Effective monitoring and surveillance are keys to mitigating a worst-case outcome in which a myopic investor suddenly discovers that money is no longer available to pay for critical expenses. Paradoxically, smaller-sized portfolios may have the greatest need for sophisticated surveillance and economic analysis because of the potentially grave economic consequences of significant declines in investment principal. However, there are also demonstrable benefits to larger portfolios that may be tasked with ambitious funding objectives including intergenerational wealth transfer and management goals.[6]

The following fact pattern may be helpful as a point of reference. Assume an investor who is a widow age 70 and in good health. She wants to enjoy a reasonable standard of living and, in addition to non-portfolio income—e.g., Social Security—requires a minimum, inflation-adjusted income of $5,150 per month. She owns a diversified $1 million portfolio allocated 70 percent to domestic and international stocks and 30 percent to domestic and international bonds. An essential point is that the target monthly cash flow represents a minimum threshold income. The investor deems that an inability to produce the periodic target income represents a personal economic catastrophe.[7] Obviously, determination of the minimum required periodic cash flow differs across the population of retired investors. Looking at the general population of U.S. retirees, a $5,150 monthly target portfolio income withdrawal might seem like an unattainable dream to many. However, the population of retirees differs from the population of retired investors because most retirees own a miniscule amount of financial assets. The case study example is appropriate because it is characteristic of the population of investors that engages the services of investment advisors who, in turn, must provide their clients with credible investment counsel.

To continue the fact pattern, two years ago, the market value of her assets was $1.25 million, and the monthly pre-tax withdrawal was $5,000 or $60,000 annually. The withdrawal equaled 4.8% of portfolio value. Today, the market value of assets is a nominal $1 million—a 20% decrease due to both withdrawals and adverse market movements—and the current pre-tax withdrawal is $61,800, the constant dollar equivalent of $60,000 at a 1.5% annual realized inflation rate. Absent fees and taxes, the total annual outflow equals 6.2% of portfolio value. Initial simulation output from a risk model indicated that the probability of success in sustaining the lifelong target income was approximately 86%. The original probability assessment suggests a 14% chance of needing mid-course adjustments which, depending on inflation and investment realizations, may be either minor or substantial. The fact pattern assumes, however, that the target income is the threshold amount required to maintain a minimum acceptable standard of living. Thus, in the investor's eyes, there may be no such thing as a minor adjustment. The

6 Family Trust instruments often direct trustees to utilize a finite amount of capital to support the income needs of a current beneficiary—often a surviving spouse—while providing funds for the remainder beneficiary class. The Uniform Prudent Investor Act [comments to §2 (a) through (d)] establishes a duty to monitor: "Managing embraces monitoring, that is, the trustee's continuing responsibility for oversight of the suitability of investments already made as well as the trustee's decisions respecting new investments." The authors of the CFA Level III course text [Arnott Robert D., Burns, Terence E., Plaxco Lisa and & Moore, Philip, "Chapter 11: Monitoring and Rebalancing," Managing Investment Portfolios: A Dynamic Process, 3rd Ed., edited by John L. Maginn, Donald L. Tuttle, Jerold E. Pinto and Dennis W. McLeavey (John Wiley & Sons, 2007)] echo this language: "Only by systematic monitoring can a fiduciary secure an informed view of the appropriateness and suitability of a portfolio for a client." Collins, Patrick J., Fast, Steven M. and Schuyler, Laura A., "Well-Performing Portfolios and Well-Disguised Insolvency," ALI-CLE Course of Study Materials: Representing Estate and Trust Beneficiaries and Fiduciaries (Chicago, 2014), pp. 499-552, provide a more complete discussion of this topic within the estate and trust context.

7 One might speculate that the investor's threshold income target reflects a standard of living shaped by habit formation; or, that she has non-discretionary expenses due to uninsured medical care costs. However the minimum income level is determined, it represents an amount required to fund the lowest *tolerable* standard of living. This should not be equated with either a *subsistence* level standard of living or an *aspirational* standard of living.

fact pattern presents a portfolio operating in a high-stress environment. Rerunning the risk model today indicates that there is an approximately 22% chance that the investor's minimum threshold withdrawals will see her outliving available resources.[8] How reliable are these risk of ruin / shortfall probability estimates? How close is the portfolio to reaching a wealth level that makes financial success unlikely? Is it prudent to "stay the course"? What planning options are available and how should the investment advisor communicate them? This constitutes the base case, and future sections of this paper revisit this fact pattern.

The remainder of the paper reflects the following organization: Part One introduces the components of a retirement income portfolio risk monitoring system. Section II provides an overview of how risk models help advisors assess a portfolio's ability to sustain adequate cash flow throughout retirement. We discuss how changes in model structure or in the value of input variables yield drastically different outputs and, hence, can produce widely varying assessments of a portfolio's economic condition. Section III presents the mathematical concept of a Free Boundary problem and outlines the advantages of viewing a portfolio monitoring system in terms of estimating the distance of current financial wealth from the wealth level—boundary—at which assets become insufficient to finance future retirement income. This is the point at which the current market value of retirement assets is less than the present value of retirement income liabilities. Section III also distinguishes between the concepts of "sustainability" and "feasibility" and argues that an effective monitoring system provides critical information about both portfolio attributes. Section IV focuses on the benefit of using a single premium immediate life annuity as an actuarial tool for present valuing the cost of the retirement income liability. If the current market value of the portfolio is greater than the current cost of an annuity designed to produce the target nominal or inflation-adjusted income, then retirement income goals remain feasible. Liability present value is, therefore, calculated under an actuarial benchmark. We remain neutral on the question of whether acquisition of an annuity contract will place any retired investor on an optimal consumption path, and we acknowledge—and briefly discuss—that there are a number of academic studies arguing for and against the strategy of annuitizing all or part of retirement assets. Specifically, we are interested in using the cost for an immediate single-premium annuity as the proxy for the free boundary. Funding a lifetime annuity income requires an irrevocable transfer of wealth to a profit-seeking insurance company. Thus, we are further interested in estimating the decrement to financial wealth that occurs by virtue of purchasing a contract that is not actuarially fair. Annuity contracts embed a host of fees, taxes, costs, and charges that, in the aggregate, constitute a "load" that must be deducted from current wealth. It is the premium that an investor pays to the insurance company in order to transfer longevity risk and to secure a guarantee of a safe and sustainable lifetime income.[9] In addition to estimating the current location of the free boundary, the section discusses how the boundary's location changes over time, and how changes in annuity pricing factors over time present complex issues for effective retirement portfolio monitoring.

In Part II, Section V illustrates how an integrated portfolio monitoring system works, the insights that it produces, and its value as an effective tool for ongoing portfolio management. It considers how a comprehensive monitoring system incorporates stochastic changes in interest rates, discount rates, longevity, inflation and other factors to provide a coherent presentation of asset management risks and opportunities. Section VI discusses how a monitoring system assists the investor to evaluate a spectrum of available asset management elections. Sections VII and VIII break new ground in the area of retirement portfolio monitoring. Section VII introduces an important risk metric: The Steady-State Ratio which measures the feasibility of providing an inheritance in addition to funding lifetime income needs. This section illustrates the costs of preserving a bequest option at various ages and under various economic conditions. Section VIII revisits the concept of investor utility. It contrasts

8 Stated differently, the model indicates a 22% likelihood that the investor will be forced to choose between an unacceptably low level of retirement consumption or eventual portfolio depletion.

9 More accurately, an annuity purchase exchanges longevity risk for (1) counterparty risk—the risk that the insurance carrier cannot pay the promised future benefits, (2) for liquidity risk, and (3) for brevity risk—the risk that an adverse change in health will unexpectedly shorten the expected planning horizon.

various utility of wealth functions and demonstrates how a withdrawal formula that may be well-suited for one investor may be ill-suited for another despite the fact that they seem to have similar goals. It argues that the ability to model and monitor hybrid distribution elections—e.g., fixed monthly amount plus percentage of portfolio, or high withdrawals early in retirement reduced over subsequent years—is an important pre-requisite to matching portfolio administration with an investor's consumption preferences. Monitoring and reporting portfolio performance for an investor preferring a front-loaded retirement income stream ("Fisher Utility") differs significantly from monitoring and reporting performance for an investor exhibiting Constant Relative Risk Aversion. The minimum periodic target income may change as a function of age. This implies a form state preference utility and demands a new approach to portfolio monitoring and management. Section IX concludes.

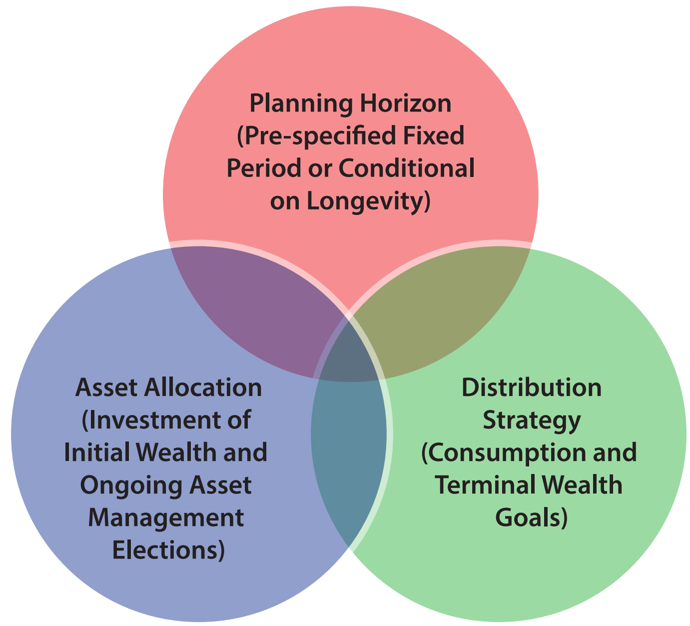
The Appendix presents a brief discussion of how annuities are priced in our retirement income risk modeling system. Details of the risk modelling system used in this essay are available at the following website address: http://schultzcollins.com/insight/wealthcaster-risk-modeling-system/.

II. Retirement Income Model Risk: Sustainability v. Feasibility

Sustainability of adequate lifetime income is a critical portfolio objective for retired investors. Commentators often define sustainability in terms of (1) an investor's ability to continue to make withdrawals throughout the applicable planning horizon, or (2) a portfolio's ability to fund a minimum level of target income at every interval during the planning horizon. The first approach focuses on the likelihood of ending with positive wealth, or, if wealth is depleted prior to the end of the planning horizon, on the magnitude and duration of the shortfall; the second focuses on the likelihood of consistently meeting all period-by-period minimum cash flow requirements.

Risk models help advisors assess a portfolio's ability to provide adequate cash flow throughout retirement. Conclusions about cash flow sustainability are usually reached by determining the likelihood that withdrawals (fixed amounts, percentage of corpus, or "dynamic") can be maintained for either deterministic or stochastic time periods under various asset allocations and longevity assumptions. Expressed in terms of a Venn diagram, portfolio success lies at the intersection of the three elements in in Figure One:

Figure One



'Sustainability' differs from the concept of 'feasibility.' Feasibility depends on an actuarial calculation to determine if a retirement income portfolio is technically solvent—i.e., current market value of assets equals or exceeds the stochastic present value of lifetime cash-flow liabilities. If the current market value of assets is less than the cost of a lifetime annuity income stream, the targeted periodic withdrawals are greater than the resources available to fund them. The portfolio violates the feasibility condition. Determination of the feasibility of retirement income objectives is not subject to model risk because the determination rests on current observables—annuity cost vs. asset value—rather than on projections of financial asset evolutions, inflation, and longevity. Prudent portfolio surveillance and monitoring tracks both risk metrics.

There are a number of modeling approaches to ascertain the likelihood that a portfolio's investment strategy is suitable to its cash flow requirements—i.e., estimating the probability that a jointly determined asset allocation / retirement spending strategy is sustainable throughout the planning horizon absent significant, and possibly difficult, mid-course corrections:

- Analytic formulae (closed form solutions usually within a life-cycle model context)
- Historical back testing of empirical returns
- Bootstrapping (reshuffled historical returns)
- Monte Carlo simulation (assuming a two-parameter normal or lognormal distribution)
- Simulating non-normal distributions (student's t, Pareto, truncated Levy flight, gamma, logistic, exponential, etc.)
- Vector autoregression
- Regime-switching simulation models.

Given the number of modeling approaches, it should not be surprising to find that there is a correspondingly wide range of model outputs. Often, the investor, or advisor, may base decision making on the output from only one type of modeling approach; and, furthermore, may not realize that even this single-perspective view of retirement risk may flow from a model that incorporates over-simplified assumptions regarding critical factors such as inflation, investment costs, and rebalance frequency. Here is the critical point: variations in a model's mathematical structure and input assumptions can lead to outputs suggesting drastically different conclusions regarding the suitability of current asset management policies to a client's financial objectives. For example, even within the simplistic Rolling Period Analysis approach, using historical returns spanning different time periods or varying the size of the rolling window can generate strikingly different success or failure probabilities. Understanding such sensitivities is essential to discerning the trustworthiness of outputs from a retirement income risk model. Although no investment risk model can predict the future, one hallmark of a credible model is that it enables investors to make good decisions within a wide range of possible futures.[10]

Econometricians often discuss model risk in terms of specification error. Errors may arise as a result of including irrelevant variables in the model, failure to incorporate relevant variables, and inaccurate estimation of input variable values. Specification errors may lead to models producing different outputs even when considering the same problem and using the same inputs. This is an underlying reason why any single retirement income risk model may be unable to provide a good assessment of retirement risk.[11]

A model is an imperfect representation of a more complex reality.[12] In this case, there are (at least) two embedded 'model risks' to consider:

1. Investors are interested in forecasts of a price change process. However, the time series of asset prices is not statistically stationary (i.e., exhibits the potential for infinite variance). It is only by differentiating the logarithm of prices on a period-by-period basis that the creation of a stationary series of returns is possible. That is to say, it is only possible to model asset *returns*; but an investor measures wealth based on asset *prices*. This is a subtle but important distinction. Returns are based on the single historical path of price changes, the realization of which is merely a manifestation of an unknown price generating process. Simulation analysis greatly broadens

10 Thomas J. Sargent's Noble Prize winning research deals with how investors make decisions when they doubt the accuracy of their model. When confronted with ambiguity, they tend to use a family of models and to over-weight bad outcomes as a mechanism for exercising caution. See, for example, Hansen, Lars Peter & Sargent, Thomas J., Robustness, Princeton University Press (2008).

11 The Society of Actuaries and The Actuarial Foundation review of a cross-section of financial planning software concludes "…programs vary considerably regarding when the user runs out of assets, if at all. Because of this finding, the study recommends that people run multiple programs, use multiple scenarios within programs, and rerun the programs every few years to reassess their financial position." Turner, John A., and Witte, Hazel A., Retirement Planning Software and Post-Retirement Risks (Society of Actuaries, 2009), p. 20.

12 Silver, Nate, The Signal and the Noise, Penguin Press (New York, 2012), p. 230 - 231, quotes the statistician George E.P. Box: "all models are wrong, but some models are useful." Silver concludes "The key is in remembering that a model is a tool to help us understand the complexities of the universe, and never a substitute for the universe itself."

our perspective about possible future outcomes; but any model of such a process must remain only a crude approximation of reality. Indeed, calculation of investment return is a function of the measurement interval (yearly, monthly, daily, intraday, continuous time) and, at the limit, may be meaningless in a statistical context.

2. The single historically realized return path for each asset class may be 'representative' of the unknown price generating process; or, may merely be an outlier result unlikely to persist. For example, an asset allocation tilt towards small and value stocks is justified based on historical return data. If the premium for investing in small and value stocks reflects a reward for systematic risks, then investors have some confidence that they will continue to be rewarded for making these investments. If, however, the premium for such investments is merely an artifact of a chance historical price change process, then investors may be increasing risk as they move their asset allocation deeper into the small/value gradient. Furthermore, investment volatility is measured by the variance statistic—the squared difference between actual returns and average return. But if the historical return path is not representative, then the concept of average becomes meaningless and statistical measures are not illuminating.

Beyond this, a savvy investor should be aware of the limitations of basing decisions on the outcome of a risk model. If optimal decisions are model-dependent, how does an investor make the best decision when the outputs of various models differ significantly? This means that the investor must consider both the credibility of each risk model as well as the economic consequences of the various choices that the risk models present. Investors are rewarded for taking prudent and calculated risks. Investors may use historical data to make inferences concerning the interrelationships between asset allocation, risk and reward. However, in designing and implementing a portfolio, it is always wise to remain aware of uncertainties in both data and the risk models that incorporate it. Past performance is not a guarantee of future results.

Monitoring involves tracking portfolio performance under various risk metrics. One commonly used set of metrics is Shortfall Probability.[13] As stated earlier, this is a forward-looking set of risk metrics that is a function of outputs from the retirement income risk model. Given the current level of financial assets, what is the possible distribution of future outcomes; and, in what percentage of trials does the portfolio meet investor objectives? What is the coefficient of retirement success? But, as noted, it is the risk model that generates the distribution of future results; and, therefore, probability assessments are not independent of the model.[14] These observations indicate that effective portfolio monitoring is multidimensional and encompasses an evaluative process which requires tracking numerous risk metrics. This is a primary reason for designing and implementing a credible retirement income portfolio monitoring system focused on both sustainability and feasibility risk metrics.

III. Retirement as a Free Boundary Problem

This section initiates a discussion on how to monitor a retirement income portfolio with a requisite degree of care, skill and caution. It does not concern itself with a hunt for the best retirement income strategies. Rather, having selected a jointly-determined asset allocation / spending strategy, the investor is concerned to verify that it remains prudent and suitable in light of financial objectives.

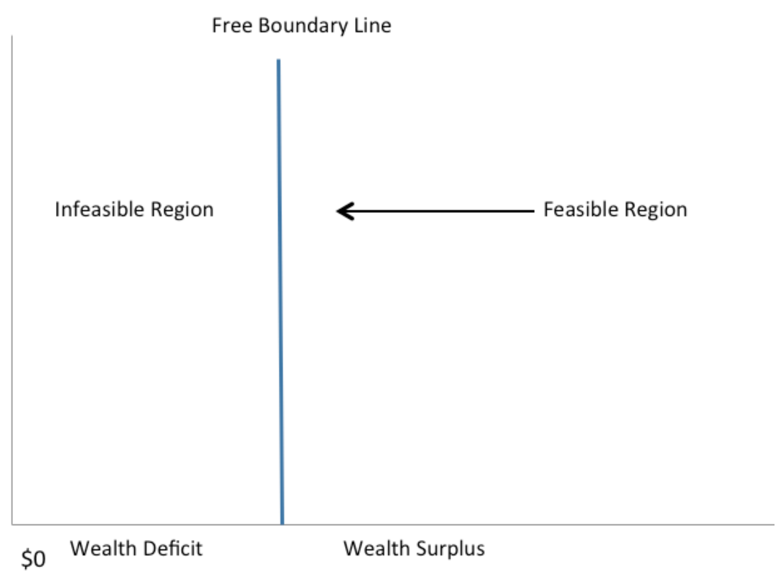
13 This set of metrics includes a variety of risk measures including the likelihood of a shortfall in either periodic consumption or terminal wealth; an unacceptable variance in periodic consumption; the expected value of a shortfall either in terms of the Present Value of a shortfall or in terms of the mean expected shortfall given that a shortfall occurs; the magnitude or duration (time without funds) of a shortfall, a distribution of shortfall results, etc. A research monograph sponsored by the Society of Actuaries' Pension Section and Pension Section Research Committee [Measures of Retirement Benefit Adequacy: Which, Why, for Whom, and How Much?] Bajtelsmit, Vickie, Rappaport, Anna and Foster, LeAndra (2013)] provides a helpful review of risk metrics.

14 Albrecht, Peter, Maurer, Raimond and Ruckpaul, Ulla, "Shortfall-Risks of Stocks in the Long Run," Financial Markets and Portfolio Management vol. 15 no. 4 (2001), pp. 481 – 499, provide a detailed assessment of shortfall risk measures. They conclude: "…the use of the shortfall probability alone is insufficient for the assessment of the risk of stock investments in the long run….the probability of a loss or a shortfall decreases with the length of the time horizon. However, the average level of the loss or the shortfall respectively, given a loss or a shortfall has occurred, increases."

A critical distinction must be drawn between sustainability—the probability calculated by the risk model that future financial market returns might be sufficient to defease the cash flow targets, and feasibility—the ability of current portfolio to produce the required cash flows. The feasibility condition requires that the current market value of assets equals the stochastic value of the lifetime target income plus, if relevant, any bequests. Conceptually, it is useful to consider portfolio monitoring as an activity within 'free boundary' problems. One class of free boundary problems, known as Stefan problems, involves estimating the demarcation between two regions where the line of demarcation is not fixed.[15] A classic example is estimating the location of the boundary between solid ice and liquid water when the temperature drops below freezing. In winter, the depth of a Minnesota lake's boundary between ice and water fluctuates according to the random variable of water temperature. In cold weather, the ice pack is thick; in warmer weather, it thins. Analogizing to retirement income portfolio monitoring, the investor faces the problem of determining the line of demarcation between two regions—a region of wealth surplus, in which the portfolio's current value is able to support financial objectives; and a region of wealth deficit, in which the portfolio's current value is not able to support financial objectives. We call the first region the "feasible region" and the second region the "infeasible region." A bull market tends to move a portfolio farther into the feasible region—the region of wealth surplus. A bear market, however, tends to move it toward the line of demarcation—the free boundary—that separates the regions. As wealth depletion pushes farther and farther toward the region of infeasibility, the consequences of investor actions or inactions are magnified in the sense that an ill-considered asset management election may not only generate losses, but losses from which the portfolio can never recover.

The investor has to know, in effect, the thickness of the ice pack, lest undue optimism induces the fatal mistake of conducting financial affairs on thin ice. Letting a portfolio continue on a path which may lead to a break in the ice is a dangerous strategy. Once the ice cracks and one falls through and is unable to climb out, no matter what the temperature is on the next day, the disaster is irreversible. If a portfolio becomes too depleted, even spectacular subsequent market performance is of no use.

The following graph presents the geometry of the free boundary concept as a portfolio moves from the region of wealth surplus toward the region of wealth deficit:



The boundary is "free" in the sense that (1) there is not a fixed dollar value that can act as the minimum required lower bound for all investors, and, (2) for any given investor, the location of the boundary constantly changes due to complex interactions between investment returns, inflation, and the discount rate used to value liabilities. It is the ratio of wealth to investment costs and spending demands that determines the boundary's location. This

15 Friedman, Avner, "Free Boundary Problems in Science and Technology," Notices of the American Mathematical Society (September, 2000), p. 854. Free boundary problems require solutions to differential equations. An introduction to these equations within the context of trust portfolio management is provided by Collins, Patrick J, and Stampfli, Josh, "Promises and Pitfalls of Total Return Trusts," ACTEC Journal vol. 27 (Winter, 2001), pp. 205 - 219. An interesting extension of free boundary problems involves the mathematics of black holes where the boundary location between normal space and the event horizon—the event horizon is the boundary at which light is no longer able to escape, so anything that penetrates the horizon is forever trapped—is both constantly shifting and unobservable. Passing through the event horizon is catastrophic. An interesting aspect of the event horizon is that you can pass through it without realizing it. Likewise, you could place yourself into what you believe to be a stable orbit only to spiral quickly into a black hole due to the smallest of perturbations. These non-linear physical phenomena provide interesting perspectives into the portfolio risks discussed in this essay. For further insight into what happens when you encounter a black hole, see the University of Colorado website: http://jila.colorado.edu/~ajsh/insidebh/schw.html.

ratio differs for each portfolio.[16] The boundary location also changes constantly as a function of investor aging, health changes, and other factors. In a bear market, the interaction of investment results with these factors tends to push portfolio value towards the boundary at either a slow velocity or, in the event of a perfect storm, at a rapid velocity. The free boundary, however, continues to be the point which determines the 'feasibility condition'—the dollar value that separates the prospect for a successful financial outcome from the prospect for a painful standard of living adjustment or, worst case, for portfolio failure and investor ruin.

Although penetrating the boundary is not an event that generates an explicit signal—there may be many thousands of dollars remaining in the portfolio at the time the boundary is breached—it nevertheless is an event that the investor should not take lightly. In terms of portfolio management, it is perhaps the single most critical piece of information that the investor should know. The existence of the feasibility condition puts a premium on intelligent monitoring. It is crucial to know how likely it is that even a one standard deviation move to the downside of the forecasted mean return could prove to be an economically non-survivable event. The investor needs to know whether they are in trouble, not whether their equity position has outperformed the S&P 500.

In terms of monitoring, we suggest that investors consider the following questions:

- Where is the free boundary line or, equivalently, what is the current dollar value of the free boundary line?
- How far is the portfolio's current value from the line?
- Has risk tolerance changed with the change in portfolio dollar value?
- What is the probability that the portfolio will move into the region of economic infeasibility?
- If portfolio value continues to move toward the free boundary, is it prudent for the investor to continue a "disciplined investment strategy"—i.e., stay-the-course?
- If a portfolio drifts into the region of infeasibility, what is the probability that wealth will rebound and return, at a future date, to the region of feasibility?
- Is an asset management strategy based on hope for such a rebound—"trust the market"—prudent?

Risk that is vague and ill-defined is not conducive to effective decision making. A monitoring system that clearly illustrates ranges of probable outcomes along a decision path can greatly facilitate the selection and implementation of appropriate asset management elections. Reference to the free boundary may be the clearest way to depict the portfolio's current economic condition.

Assume a modest-sized portfolio is losing value in a bear market. What is the likelihood that it will be unable to discharge its financial objectives? The answer to this question quantifies "bankruptcy risk." In our example, bankruptcy risk is (1) the *probability* of portfolio depletion while the investor remains alive, and (2) the *magnitude* of the shortfall—the amount of time the investor spends without sufficient financial resources; or, time alive-but-broke. Bankruptcy risk focuses on the likelihood of portfolio depletion in the future. Shortfall risk, a complementary risk metric, focuses on the likelihood that the investor will be unable to maintain threshold income/bequest targets. The portfolio can continue to provide income; but, the level of income is inadequate to support investor needs. The wealth / income targets might be expressed in aggregate dollar terms—i.e., a floor value below which the investor does not desire to penetrate; or, in periodic income terms—a minimum income that must be available each and every month during the applicable planning horizon. As shortfall risk increases, a monitoring system can anticipate problems and can help determine what corrective actions can be taken today lest the portfolio suffer a catastrophic failure in the future.

A related question asks what current market value is equal to or greater than the present value of the future

16 The assumption here is that the investor has specified the required minimum or lower-bound periodic withdrawal amount, irrespective of whether it is expressed in nominal or constant dollar terms. This is the amount that the portfolio must provide lest the investor experience a catastrophic economic result. Thus, the demands on the portfolio are net of any other sources of income such as Social Security or employer pension benefits.

cash flow liabilities. This is a *solvency* question; or, in terms of the free boundary problem, it is the amount of wealth that separates the region of feasibility from the region of infeasibility. It is not a future-oriented prediction; it is a question of the adequacy of current resources. Determining shortfall risk and quantifying solvency by locating the free boundary are both important components of risk assessment, and both provide important feedback regarding asset management dangers and opportunities.

The boundary's location constantly shifts, typically receding as investment returns become positive and drawing closer as investment returns become negative. Deterioration in investor health pushes the boundary away; a cure for a mortality-impairing ailment pushes the boundary closer. A lower or higher than expected inflation rate may also influence the location of the free boundary. A portfolio, the value of which falls below the free boundary, may reenter the region of feasibility as a result of a change in investor circumstances or as a result of a strong market recovery. However, falling below the boundary implies that, at the present time, the portfolio is technically insolvent—it lacks sufficient money to fund the investor's threshold consumption target throughout the planning horizon. Falling below the boundary does not mean the portfolio is broke—its current dollar value may be substantial. However, relative to its minimum or lower-bound distribution target, it has violated the "feasibility condition": assets are less than the present value of liabilities.

Although the feasibility condition [assets ≥ liabilities] is conceptually easy to understand, nevertheless it is difficult to track. Each portfolio has its asset allocation, fee structure, distribution target, tax liabilities, and so forth. It is easy to see that the current value of assets constantly changes as investment results are realized. In a single week, month or quarter, the fair market value may rocket up or down depending on the volatility of investments. Likewise, changes in inflation and interest rates will change the discount rate for valuation of future cash payments; life expectancy changes can increase or decrease the planning horizon. Indeed, the planning horizon—and thus the present value of the aggregate liability—changes merely with the passage of time.

A portfolio remains in the region of feasibility when:

The Present Value of Assets ≥ Stochastic Present Value of Distributions + Stochastic Present Value of Bequests + Stochastic Present Value of Fees and Investment Expenses.

The free boundary for the purposes here is located at the exact point where the present value of assets equals the stochastic present value of distributions required to fund the investor's threshold cash flow requirements. Of course, the amount by which assets exceed liabilities equals a cushion or investment surplus.

At any point in time:

Total Portfolio Wealth = The share to be distributed to fund a threshold standard of living + The share to be bequeathed + Taxes, fees and other costs[17].

For an income-oriented investor, the bequest amount is the economic equivalent of an investment surplus. The term 'withdrawal surplus' is also helpful because it highlights the concept that as long as the bequest amount is positive, there are sufficient assets to pay the projected threshold income target. As the surplus shrinks, the risk that the portfolio will be unable to fund threshold needs increases. Retirement portfolio management may be defined as a contest between consumption and bequest goals. As the surplus shrinks, *risk to the periodic income stream and to terminal wealth increases at an increasing rate*.

Assume a $6 million portfolio with a $3 million estimate of the Present Value of the spending liability. If we consider the Wealth/Surplus ratio defined as: (Wealth) ÷ (Wealth – PV Consumption), the current ratio value is $6 million total assets ÷ $ 3 million surplus = 2.[18] At this wealth level, a 1% change in the value of assets translates in an approximately 2% change in the value of

17 The sign of the last right-hand term is usually negative although certain tax credits may impact the sign.

18 The ratio is actually a ratio of two distributions rather than two numbers. The numerator reflects the probable range of short-term asset values, and the denominator represents Wealth minus the probable range of short-term changes in the Present Value of liabilities. The current value of assets is a reasonable proxy for the numerator, and the mean of the liability distribution [50th percentile of aggregate distributions discounted for inflation] is a reasonable proxy for the denominator. The ratio does not consider fees and investment charges which would represent a further adjustment in the denominator's value.

the portfolio's investment cushion. Assume, further, a 25% decrease in the value of assets to $4.5 million. The ratio is now 4.5 ÷ 1.5 = 3. This translates into a change in the value of the investment surplus of approximately 3% for each 1% change in the value of assets. Finally, assume a further decrease in portfolio value to $3.5 million. The ratio value is now 3.5 ÷ .5 = 7. This is a "leverage factor" of seven to one! As the investment surplus disappears, does the investor stay-the-course with respect to the asset allocation? Investor risk tolerance is almost certainly changing as wealth decreases. Tracking the changes in the value of the wealth-to-surplus ratio is an important aspect of an appropriate portfolio monitoring program.[19]

Whenever a portfolio is in a pure accumulation stage—no cash flow into or from the portfolio—there is a presumption that, absent significant changes in investor circumstances, investment policy should remain largely static. An asset-only management perspective, however, may not be optimal for retirement income portfolios. The CFA Institute, for example, acknowledges that "asset allocation policies are likely to change over time as characteristics of the investor change and as market circumstances vary" and that "…volatility as a descriptive measure of risk may be irrelevant beyond an absolute level of loss that would completely derail an investment portfolio."[20] Reference to "…an absolute level of loss that would completely derail an investment portfolio…" is a good expression for passing into the region of infeasibility.

In an Asset-Liability portfolio management context, it is the distance from the free boundary that is a key measure of portfolio risk. The investor must closely monitor the dynamic interactions among asset price changes, the strategic asset allocation, and the value of the retirement cash flow liability. Paradoxically, an asset management plan that calls for periodic rebalancing to the strategic asset allocation target may not be the economic equivalent to a stay-the-course approach. For portfolios making periodic withdrawals in a decreasing market, portfolio risk emphatically does not "stay the same." As current wealth declines, some investors put a premium on strategies to increase future expected return. The sooner a portfolio recovers investment losses, the happier all interested parties. However, strategies calling for higher growth portfolios are more volatile. While the higher expected growth may reverse investment losses, the higher volatility also increases the probability that wealth will continue to decline. There is a risk to increasing portfolio risk. At the limit, if the only response to losses is to seek investment gains by increasing risk, asset management becomes mere gambling. Return and variance move together, and any attempt to increase return also increases the probability of failure. Losses do not mandate increasing risk in order to recoup the losses.

IV. The Free Boundary Location

How do you determine the free boundary location—i.e., the "level of loss that would completely derail an investment portfolio"? The present value of threshold lifetime income for the retired investor is an actuarial calculation. Therefore, one measure of required wealth is the expected price of a single premium immediate annuity generating an income stream sufficient to meet the minimum spending target. If there is not sufficient wealth to purchase an annuity,[21] there is a danger that the portfolio cannot sustain the minimum income over the investor's remaining lifespan. Annuity prices are an observable, market-based proxy for the cost of providing a lifetime income.

Annuity cost benchmarks present several challenges for the design and implementation of a retirement income risk model. A useful model is forward looking in that it provides insight into both the expected values of critical variables and the extent to which actual results may differ from expectations. Just as there is a probability

19 Equivalently, if Wealth is the numerator and Consumption is the denominator, the monitoring system will want to track the Wealth/Consumption Ratio. If using a Wealth/Surplus Ratio, the lower the ratio value, the better a portfolio is able to fund future liabilities. By contrast, a high Wealth/Consumption ratio value suggests manageable demands against financial resources; a low ratio value suggests that adverse market conditions might stress the portfolio to the point where it can no longer be expected to generate the threshold income.

20 Elements of an Investment Policy Statement for Individual Investors CFA Institute (2010), §3c., p. 6.

21 An annuity is defined as a single-premium, immediate life annuity with no period certain or refund features. See Dus, Ivica, Maurer, Raimond and Mitchell, Olivia S., "Betting on Death and Capital Markets in Retirement: a Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans," University of Michigan Retirement Research Center (RB 2005 – 053) for a discussion of five types of annuities that fit the general definition of a single premium immediate life annuity.

distribution of investment returns, inflation rates, and investor lifespan; so, also, is there a distribution of future annuity costs. Annuity prices are neither constant nor fully predictable from month-to-month; and, therefore, the cost of discharging the liability to provide future income is also a random variable. Annuity prices are stochastic and are a function of several factors including the interest rate at the time of purchase, longevity expectations for the annuitant, and the explicit and implicit costs of the annuity contract. The risk model must present an expectation of future annuity costs as well as a more precise current cost estimate. An expectation is a future-oriented value—it is a value that is more-likely-than-not to be close to what is realized; an estimate is a cost based on specific current data—it is a value that reflects the actual economic and demographic conditions of the day. In terms of the previous discussion of model risk, the expected future annuity cost projection is a reasonable guideline for making long-term asset management decisions; the current price of an annuity is a measure of variance (risk) in the cost projection. *A priori*, we expect an increase in the magnitude of the difference between actual and expected during periods of abnormally low or high interest rates.

An actuarially fair annuity is the sum of the mortality-adjusted payments where each payment is discounted by an appropriate interest rate. Unfortunately, an actuarially fair annuity cannot be purchased. Commercial annuities have both explicit costs such as premium tax, contract fees, sales loads, etc., and implicit costs reflective of insurance company profit targets, reserving requirements, tax liabilities, and cost-of-capital hurdle rates. Therefore, if an annuity contract is an appropriate benchmark for determining the feasibility condition, the issue reduces to: (1) what is the threshold income required to sustain a retired investor's minimally acceptable standard of living; and, (2) how much current financial wealth must be ceded to an insurer to secure the requisite threshold income stream? If current wealth is below the current cost of funding the minimum lifestyle target income, the portfolio violates the feasibility condition.

In one sense, finding the current price of an annuity is easy. Several websites allow the user to enter the requisite amount of monthly income as well as other contract features including graded payment increases in order to determine the contract's cost. Additionally, major custodians [e.g., Fidelity and TDAmeritrade] as well as mutual fund companies [e.g., Vanguard] provide prices for a select range of annuity contracts. The current annuity price determines the ratio of portfolio value to annuity cost. Thus, if an investor currently owns a $2 million portfolio and obtains a $1 million market price for an immediate annuity providing the lifetime target income, the ratio value is 2÷1 = 2, or, a 2x coverage ratio. Any ratio value below 1 signifies that current wealth is insufficient to fund the actuarial equivalent of the investor's target income stream. That is to say, the target income is not 'feasible' given current resources, current interest rates, and other market pricing factors including current age.[22]

However, an effective portfolio monitoring program will want to do more than simply present a single point-in-time annuity cost. As stated, the annuity cost is, itself, a stochastic variable. The program will want to estimate, over time, the portfolio Wealth to Annuity Cost Ratio [WACR] to provide insight into how the investor's preferred "safety margin" may dynamically unfold. For example, if the investor is committed to converting all or part of portfolio wealth to an annuity once the coverage ratio goes below, say, 1.10 then examining the projected change in ratio value over time indicates the likelihood of a future conversion from financial assets to actuarial assets. Should annuitization not prove palatable, insight into the ratio value's trend provides useful information for determining if changes to asset allocation, investment strategies, tax and asset location elections, and so forth are prudent. The WACR is a solvency benchmark; it is not a recommendation to buy an annuity contract.[23]

22 One might object that if the expected equity risk premium is higher than the annuity return, the investor can fund retirement cash flows more cheaply by maintaining exposure to risky assets. This is the case for investors with substantial surplus wealth who wish the opportunity to improve their budget constraint so that they can enjoy a higher future standard of living. More money is always better than less. However, as stated, the level of wealth relative to consumption requirements is critical in determining the lottery that an investor is willing to accept. Penetrating the free boundary established by the existence of a minimum periodic income need risks financial catastrophe, and is not a lottery that is easy to accept when acceptance is based on mere projections of future equity risk premiums. When investment returns are bad, asset management choices become difficult.

23 See, for example, Pittman, Sam and Greenshields, Rod, "Adaptive Investing" A Responsive Approach to Managing Retirement Assets," The Retirement Management Journal, Vol. 2, No. 3 (2012), pp. 45 – 54.

The demands on the portfolio monitoring system place a premium on its ability to estimate the amount of liquid financial wealth that must be traded both now—a current observable—and in the future—an estimate of expected future annuity costs—to secure the annuity's lifetime income guarantee. Although annuity costs are often a topic of interest in academic research, developing accurate cost decomposition is difficult.[24] The term "load" subsumes all costs beyond the price of an actuarially fair contract derived by applying mortality adjustments and interest rate discounting to the periodic income stream. Thus, the term "load" encompasses explicit costs such as agent commission and sales distribution system overwrites, contract set up fees, and ongoing administration charges; as well as implicit costs such as insurance carrier line-of-business profit targets, and shareholder return on investment demands.

The load is, therefore, the difference between (1) the annuity pricing factor, which itself is based on a sum of fractions where the numerator of each fraction is the periodic payment adjusted for the probability of being alive to receive it, and the denominator of each fraction is the applicable discount rate, and (2) the annuity's market price. Insurance companies differ in their calculation of the numerator—i.e., adjustments to the mortality table used to set legal reserves, as well as in their choice of discount rates for the denominator. Pricing elements depend on a variety of non-observable insurance carrier decisions. These include strategic marketing decisions such as an attempt to hedge life insurance liability exposures by issuing annuity contracts, capital structure decisions, and strategic investment decisions such as exploiting investment opportunities in the private placement bond market.[25] The price that an investor pays for an annuity equals the actuarially fair price plus an additional decrement to wealth reflecting the 'load.'

Returning to the sample fact pattern of Section II, the investment portfolio has declined in value from $1.25 million to $1.0 million. Its shortfall risk, as determined by simulation of a credible risk model, has increased from approximately 14% to 22%. Given the annual $61,800 threshold income demand [an inflation-adjusted $5,150 per month assuming two-years realized inflation at a 1.5% per annum rate], the investor is interested in knowing how close her portfolio is to reaching a wealth level that unambiguously places it in the region of infeasibility. Given that the threshold income target must be met with certainty, an appropriate proxy for determining the present value of the cash flow liability is a single premium immediate annuity contract that aligns with the investor's periodic income needs.

Calculating and projecting the actuarially fair cost of the annuity income stream requires multiplying the periodic target income times a fraction, the numerator for which is the probability of remaining alive to receive payments throughout the planning horizon, and the denominator of which is a time-value of money discount factor. Once the actuarially fair annuity pricing factor is available, multiplying it by (1 + load percentage) yields the estimated cost of the annuity contract. We estimate (calculations not shown) annuity loads to be in the 13% to 18% range.[26] Given reasonable estimates of current interest rates earned by corporate bonds and mortgage backed securities found in insurance company portfolios[27], and assuming a 15% load, the risk model calculates that a single premium immediate annuity contract providing the following target monthly lifetime

24 Donnelly, Catherine, Guillen, Montserrat and Nielsen, Jens Perch, "Bringing cost transparency to the life annuity market," Working Paper (November, 7, 2013) state "Consumers have no idea if annuity prices are fair, or if insurance companies are either making excessive profits or are grossly inefficient." Electronic copy available at: http://www.ifid.ca/Conference_Material/donnelly_IFID_Conf_2013.pdf.

25 The *cost* to manufacture an annuity contract reflects, in part, the investment opportunity set available to large insurance firms who operate in a private placement bond market. The market *price* of an annuity contract reflects, in part, the profit objectives of the manufacturing insurance company. The *value* of an annuity contract to the buyer reflects, in part, the yields available in the public bond market as well as the buyer's coefficient of longevity risk aversion. In this section, we are primarily interested in price.

26 Our findings are in line with previous studies. For example, Pang, Gaobo and Warshawsky, Mark, "Optimizing the Equity-Bond-Annuity portfolio in Retirement: The Impact of Uncertain Health Expenses," Insurance: Mathematics and Economics vol. 46 no. 1 (2010), pp. 198 – 209 set the annuity expense load to 15% in their simulation model.

27 Many bond issues are private placements which, because of discounted underwriting fees, are purchased at advantageous costs. According to LOMA, the median net portfolio yield for all life insurance companies in 2012 was 4.19%. Median yield on the bond component of portfolios was 4.31%. However, a histogram of the bond yield distribution shows the mode (highest number of companies within the interval) at 5.25% to 5.75%.

benefits to the age-70 female investor in good health[28] could be purchased for $1,000,000:

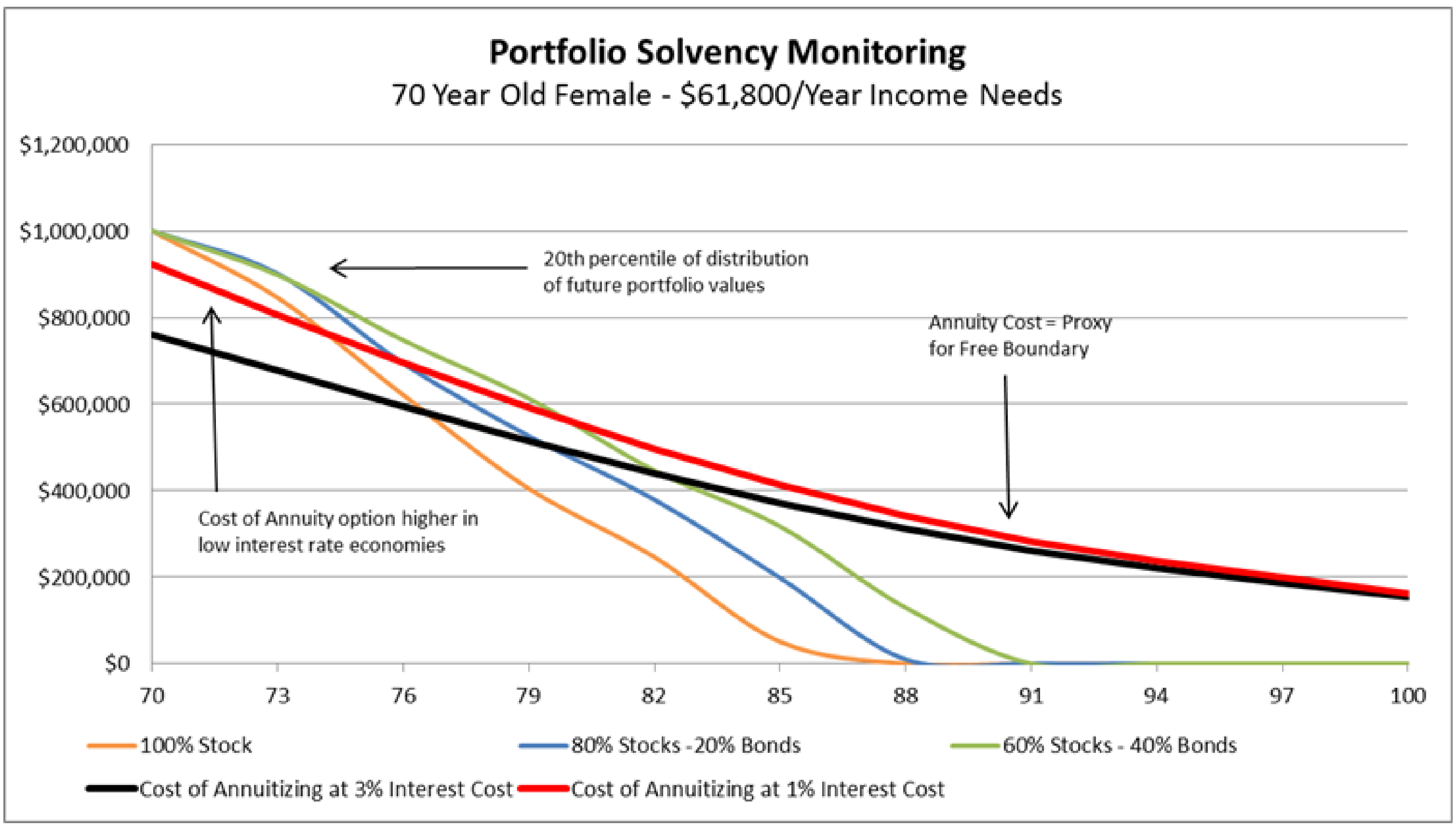
- Fixed Nominal Benefit: $6,640 per month
- 2% per year Graded Benefit: $5,676 per month

Given the monthly target income of $5,150, it appears that the investor is currently in the region of feasibility at an approximate $100,000 distance from the free boundary [$5,150 ÷$5,676 = .9073 x $1,000,000 = $907,000].

Although the distance to the free boundary is only 10% of the portfolio's current value, the annuity cost calculation occurs in a historically low interest rate environment. Annuity payouts are sensitive to changes in interest rates; and, given the mean-reversionary tendency of interest rates, there is a reasonable expectation that the wealth to annuity cost ratio may, all else equal, improve. For example, in our model's calculation of current annuity costs for a female investor age 70, a 50 basis point upwards parallel shift in the yield curve generates a 2% graded monthly benefit of $5,917, while a 100 basis point upwards parallel shift generates a 2% graded benefit of $6,162. [29]

As interest rates revert towards their historical mean, the complex interactions between possible negative effects of rising rates on the financial asset portfolio and positive effects on annuity costs will either expand or shrink the distance from the free boundary.

The following graph illustrates the impact of differing interest rate environments on a portfolio monitoring system:



28 'Impaired life' annuities are written by several insurers. These instruments offer increased monthly benefits to annuitants suffering from impairments that are expected to shorten their lifespan. See, for example, Brown, Robert L., and Scahill, Patricia L., "Issues in the Issuance of Enhanced Annuities," available on the Society Of Actuaries website at soa.org [2010]: www.soa.org/library/journals/.../apf-2007-10-brown-scahill.pdf.

29 For further insight into the sensitivity of annuity pricing to changes in interest rates, see "The Annuity Duration Puzzle," N. Charupat, M.Kamstra and Moshe S. Milevsky (2012) available at www.vgsf.ac.at/fileadmin/user_upload/PDF/seminar_archive/summer_term_2012/2012-05-04-kamstra-annuity.pdf

The graph illustrates a variety of asset management elections currently available to the investor. She could maintain the current 70-30 stock-to-bond allocation; or, could elect to increase or decrease risk in response to the recent declines in her portfolio's value. In this case, we bracket her current allocation by moving the equity exposure to 100%, 80% or 60%.

The graph provides insight into the following question: given current portfolio's dollar value, how might continued investment in a risky-asset portfolio impact the feasibility of the financial objectives? The graph portrays left-tail outcomes (the 20th percentile of results in a bull/bear regime switching model with the initial regime selected randomly).[30] It depicts constant-dollar results for the 100% stock portfolio (orange line), the 80% stock portfolio (blue line) and the 60% stock portfolio (green line). If the investment climate continues to remain unfavorable, the x-axis time line indicates portfolio depletion for the 100% and 80% stock portfolios at approximately age 88; and depletion for the 60% stock exposure portfolio at approximately age 91. Of course, the retirement income model indicates an approximately 20% chance of portfolio depletion prior to these dates. Additionally, the model suggests that trying to recapture lost portfolio value by increasing expected return through additional equity exposure is a risky asset management strategy.

Superimposed on the left-tail of the probability distribution are two free boundary lines. The red line is the cost of purchasing an annuity to provide an inflation-adjusted benefit of $61,800 per year assuming that current interest rates remain at 1% and that the shape of the pro-forma yield curve derived by the retirement income risk model does not change. The black line is the cost of purchasing an annuity to provide an inflation-adjusted benefit of $61,800 per year assuming that current interest rates rise to 3% and that there is a corresponding parallel shift in the yield curve. As the investor ages, the cost of purchasing an annuity sufficient to fund her lifetime threshold target decreases, all else equal. Indeed, the annuity's price, at any interest rate, converges to the same amount as the investor approaches age 100. At this point in time, the effect of mortality dominates the effect of interest rate exposure.

30 Details of the retirement income risk modelling system are available at the Schultz Collins website address: http://schultzcollins.com/insight/wealthcaster-risk-modeling-system/

What does the graph indicate? Currently, the portfolio value is above the free boundary—i.e., in terms of the client's minimum income objectives, the portfolio fulfills the solvency condition. Although moving towards a riskier portfolio offers the expectation of higher future returns, a change in allocation requires close monitoring. There is an approximately 20% probability that increasing the equity weighting above 80% may accelerate the risk of portfolio depletion by age 88.[31] Indeed, if interest rates remain historically low, a 100% equity, 'swing-for-the-fences' portfolio offers a 20% chance of penetrating the free boundary by approximately age 74. If interest rates rise, however, the investor has much more breathing room. The 20th percentile results do not fall below the free boundary until the late 70s or early 80s. The investor has sufficient time to delay annuitization; surplus wealth provides her with the luxury of waiting to see how the capital markets perform. ▪

31 The illustration assumes that the investor continues to withdraw the income required to maintain her threshold standard of living despite the risk of future portfolio depletion. Although investment results may force the investor to reassess her standard of living threshold, the purpose of the chart is to quantify the feasibility of her income target. The strength of bequest motives may also influence the investor's asset allocation decision.

Monitoring and Managing a Retirement Income Portfolio

PART TWO

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V. Putting it All Together: Monitoring a Retirement Income Portfolio

This section explores, by means of a case study, how a comprehensive risk modelling system provides insights critical to prudent portfolio management. It introduces the Wealth/Annuity Coverage ratio. The ratio is not a replacement either for utility-based or shortfall probability metrics; rather, it is a tool for assessing the current economic condition of a retirement portfolio.

The case study assumes a constant-dollar fixed amount withdrawal strategy. This is not to suggest that this strategy is the utility maximizing strategy, the most sustainable strategy, the strategy that produces the greatest aggregate income, the strategy with the best reward (income) to variance (fluctuations in periodic income throughout the planning horizon) tradeoff, or a strategy that dominates according to any of a large number of other preferencing criteria. Rather, we use the strategy for three primary reasons: (1) it is simple to understand, (2) we assume that the withdrawal amount is the minimum required to support a threshold standard of living—i.e., the investor has little flexibility with respect to cash flow amount and timing, and (3) it is a strategy that retirees interested in budgetary certainty often aspire to implement.

The portfolio monitoring and surveillance techniques that we discuss are, in many instances, adaptable to other withdrawal strategies. Section VIII [Hybrid Withdrawal Strategies] examines how a risk modelling system helps retiring investors compare a variety of withdrawal strategies including a percentage-of-corpus (unitrust) withdrawal; front-loaded retirement income (the Irving Fisher utility-maximizing strategy); back-loaded retirement income; and various combination—hybrid—strategies such as fixed amount plus percentage-of-corpus withdrawal formulae.

Previous research demonstrates the importance of the Wealth/Consumption ratio in determining the likelihood of an economically successful retirement.[32] Similarly, the Wealth/Annuity Cost Ratio [WACR] aggregates future consumption into an observable net present value cost as proxied by the cost of an annuity. The WACR monitors *solvency risk*. A portfolio is solvent if current assets equal or exceed the amount required to fund the target income. Following a substantial market decline a portfolio may bounce back and recover enough dollar value to mitigate shortfall risk. *Or it may not*. Shortfall risk is a function of future probabilities—not current portfolio values. A solvency risk metric, by contrast, determines how far portfolio value would have to fall so that it can no longer produce lifetime income absent a market recovery. It is the threshold point below which the investor 'hopes' for a favorable outcome rather than 'expects' one.

The concept is similar to regulatory solvency monitoring for banks and insurance companies. A firm may have a current asset portfolio worth many millions of dollars. However, if current asset value is below the solvency threshold established by regulators, the firm can no longer continue normal business operations. The risk model facilitates implementation of a solvency monitoring program by communicating the distance from this critical threshold. Under the Wealth/Annuity Cost Ratio metric, the investment advisor makes an initial determination that the current value of retirement assets meets the feasibility condition. This is equivalent to saying that the current portfolio's value is equal to or greater than the cost of providing threshold income through a commercially available annuity; or, that the ratio of current portfolio value to current annuity price ≥ 1.

If, however, the analysis determines that the initial ratio value is < 1, the retirement income withdrawal plan violates the feasibility condition—wealth is below the free

32 See, for example, Ho, Kwok, Milevsky, Moshe, and Robinson, Chris "Asset Allocation, Life Expectancy and Shortfall," Financial Services Review Volume 3, Issue 2 (Summer,1994), pp. 109 - 126.

boundary. The investor hopes that the portfolio's expected future return will eventually restore the portfolio to a value that satisfies the feasibility condition. However, the sequence of realized returns may initially be negative—a bear market—making the likelihood of future recovery remote. If the current ratio value is less than 1, the investor may be best advised to postpone exiting from the labor force, secure part-time retirement labor income, or redefine threshold income necessary to fund the minimum acceptable standard of living. Otherwise, as the title of one research study suggests, the investor is betting on death and the capital markets.[33]

The Case study unfolds with Graph One.[34] The minimum income is $5,150 per month adjusted for inflation. Although the retiree would like to have a greater amount of monthly cash flow, additional income is "aspirational" rather than "required." Furthermore, if possible, the investor would like to bequeath the original principal—in constant dollars—to her heirs.[35] The case study, at its outset, imposes a viewpoint regarding the market's initial direction. In this case, the investor believes that the return generating process will begin in a bull market economy. The case study begins in a bull market regime for expositional convenience only. If there is a strong case for initiating the return evolutions within a specific regime—perhaps because of significantly high or low market valuation indicators—the risk model can accommodate differing viewpoints. Most commonly, the retirement income risk model randomly selects the initial regime (bull or bear) without requiring the investor to bet on the correctness of market prognostications.

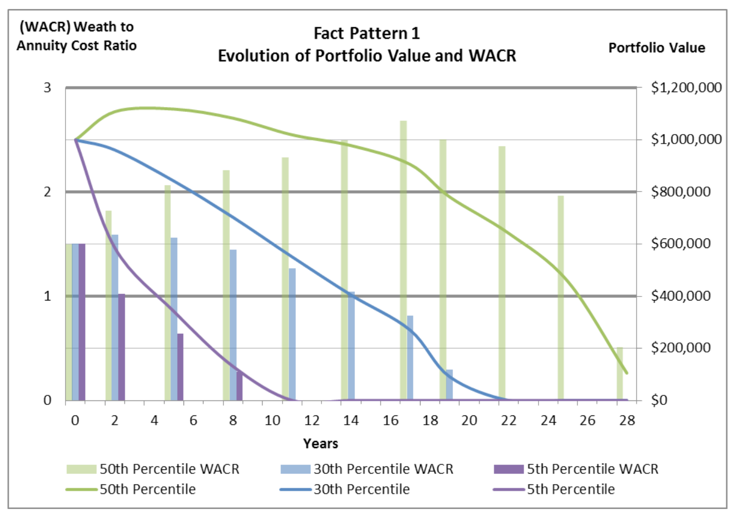
33 Dus, Ivica, Raimond, Maurer and Mitchell, Olivia S., "Betting on death and capital markets in retirement: a shortfall risk analysis of life annuities versus phased withdrawal plans," Financial Services Review Volume 14, Issue 3 (Fall, 2005), pp. 169 – 196.

34 The Assumptions underlying fact pattern #1 are similar to the base case introduced at the beginning of Part One. Fact pattern #1 incorporates a fee schedule for investment advisory services—including a minimum annual fee, and has a factor for costs incurred in maintaining a well-diversified portfolio consisting of index mutual funds and ETFs. It uses a regime switching model with the initial economic state set to "Bull market." The investor is age 70, and in good health. Her initial portfolio value equals $1 million and initial pre-tax distributions are $61,800 per year adjusted for inflation. Annuity loads decrement existing wealth by 15%.

35 The fact pattern mirrors, by design, Settlor instructions found in the governing instruments of many irrevocable family trusts: provide income sufficient to meet the current beneficiary's needs and, if possible, preserve principal for the benefit of the remaindermen.

Graph One:

Graph One depicts the evolution of constant-dollar portfolio values net of investment expenses, advisory fees, and monthly pre-tax cash distributions. The risk model generates a probability distribution of future portfolio values based on 5,000 simulated trials. It ranks results from worst case (1st percentile = lower dollar value) to best case (99th percentile = higher dollar value). The graph's left axis measures the ratio of the portfolio's dollar value to the *expected* cost of purchasing a commercial annuity providing the inflation-adjusted equivalent of the investor's initial income target.[36] Columns visually depict the evolution of this ratio at the 50th, 30th and 5th percentiles of the distribution. As long as the portfolio's WACR—measured on the left axis—stays above the horizontal line marking a ratio value of 1, there is an expectation that the investor's income goal remains feasible. The right axis measures the evolution of the portfolio's constant dollar value through time. The graph depicts constant-dollar values at the corresponding 50th, 30th and 5th percentiles.



The expected annuity cost, using a pro-forma yield curve at the investor's age 70, indicates that the portfolio's value is approximately 150% (1.5x coverage) of the cost of an inflation-adjusted annuity. This suggests that retirement

36 We distinguish between the *estimated* cost of purchasing an annuity at a given moment in time—which is largely a function of the current term structure of interest rates—and the *expected* cost of an annuity given a pro-forma yield curve described in the appendix. The median value of a simulated return series is the expected return. However, actual or realized returns vary around the expected value. The future cost of an annuity is also a random variable. A pro-forma yield curve generates the expectation; the actual interest rate environment determines the variance from the expectation.

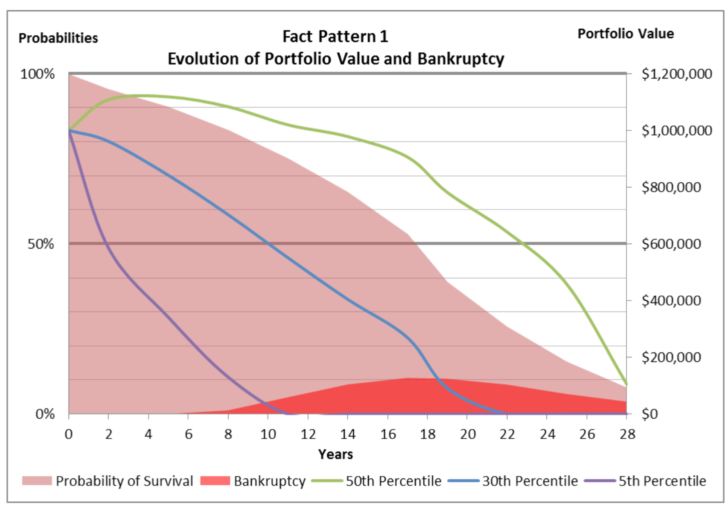
income goals are currently feasible. Under a bull market regime, there is the expectation [50th percentile of the distribution of simulated results] that she can achieve both her future income target and bequest objectives for at least the next 14 years.[37] However, even assuming an initial bull market regime, the 30th and 5th percentile of the probability distribution both suggest that she may face a future shortfall.[38] Indeed, at the 5th percentile, the magnitude of the expected shortfall may be substantial as all portfolio wealth is depleted after approximately 11 years.

As previously noted, the risk model generates expectations. The reader might recall that the WACR calculated at the end of Part One is only 1.1x coverage. In addition to setting reasonable expectations, a credible monitoring system must also track the variance around the expectation. In this case, we employ a model feature which contrasts expected annuity costs derived from a pro-forma yield curve to a more precise cost calculation based on the current yield curve environment. One element of the risk model builds a pro-forma annuity cost structure; the other element checks to see how actual current costs vary from expectations as current interest rates fall either above or below their historical mean.

This process has two benefits: (1) current optimism or pessimism regarding the cost of acquiring an annuity contract does not unduly influence investor decision making; and (2) the risk model provides insight into how mean reversion in interest rates may impact the wealth-to-annuity-cost ratio in future years. In the expected case, the WACR increases, by year 16, to an expected 2x coverage; in the worst case (5th percentile), however, the ratio value falls below 1x coverage in approximately two to three years. Stated otherwise, the retirement income risk model (1) gives the investor sufficient warning regarding the possibility of a potentially fatal blow to her retirement aspirations that may unfold within the next several years, (2) provides an indication of the speed and magnitude of the problem should the portfolio realize lower than expected returns, and (3) provides sufficient time to rethink retirement income portfolio strategies and consider alternative asset management elections should realized investment returns continue to remain below expected value. In the long run, the initial risk picture suggests that she can expect to provide a substantial inflation-adjusted bequest ($500,000 or greater) over the next 25 years. Additionally, she can expect to maintain a level of portfolio wealth sufficient to provide the inflation-adjusted lifetime target—i.e., a WACR ≥ 1—until approximately age 96.[39]

Graph Two:

This graph continues Graph One's fact pattern and presents an analysis based on a typical shortfall risk metric—the likelihood that an investor remains alive having depleted the portfolio. We use the term 'bankruptcy' to reflect the fact that no further assets remain in the portfolio. The graph assesses the probability for financial success if the asset allocation and withdrawal policies remain on autopilot. Stated otherwise, the graph indicates the probability that the investor must make mid-course corrections in order to avoid a financial catastrophe. It depicts the conditional risk of bankruptcy—alive without funds—if retirement begins in a bull market.



37 If the fact pattern failed to incorporate fees and expenses, the graph would show an expected portfolio value equal to or greater than $1 million in constant dollar terms throughout the entire planning horizon. When modeling retirement risk, investment costs matter—a lot.

38 Although the bull market distributions of investment returns for most asset classes are more favorable than corresponding bear market regime distributions, this does not guarantee a positive return. Even under bull market regimes asset class distributions exhibit left tails with large negative returns.

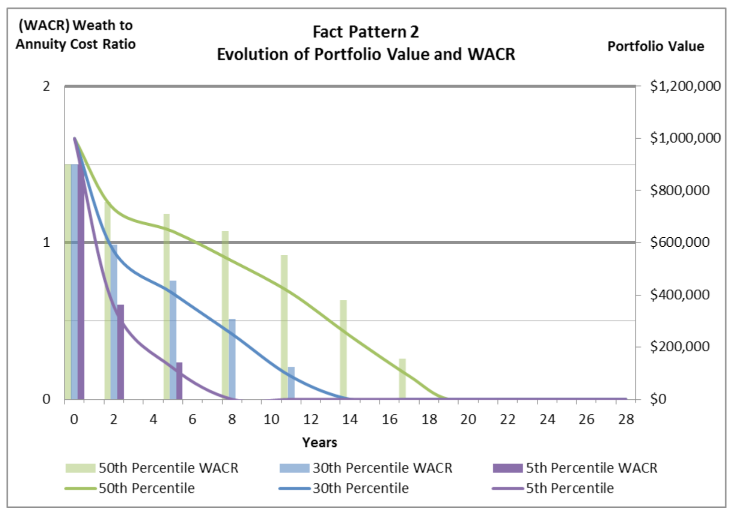
39 In today's marketplace, it is unlikely that an investor older than age 85 could purchase an annuity. However, this might change as actuarial experience with older-age annuitants expands.

The graph's left axis measures the probability of survival to year x. The light-colored, fully-shaded area depicts expected longevity based on the Society of Actuary Table for high-income, white-collar female retirees from defined benefit pension plans. Taking into account investor longevity, the graph's red-colored area indicates the conditional risk of bankruptcy. Conditional bankruptcy risk is defined as the intersection of two independent events: the chance of remaining alive at year x and the chance of a zero portfolio value in year x. Conditional bankruptcy shows a hump-shaped pattern because, as time passes, a greater percentage of surviving investors remain alive without funds; but the absolute number of surviving investors decreases due to the force of mortality.[40] The conditional bankruptcy rate at year x is the ratio of the probability of bankruptcy for surviving investors divided by the probability of survival at that time. In this case study, by year 11, the probability of survival for female investors in good health who start retirement at age 70 is approximately 75%. From the subpopulation of surviving investors, less than 7% are alive without funds [5.0 % unconditional bankruptcy rate ÷ 75.0% probability of survival = 6.6% conditional bankruptcy rate]. After twenty years the conditional bankruptcy rate rises to 1 in 4. While 1 in 4 may seem high, the tradeoff being made is to present heirs with a meaningful estate should death come earlier. As noted in the discussion of graph one, in the median case, portfolio value remains nearly stable on a CPI adjusted basis over the next 14 years. Although the risk is present, it is small and can be managed prudently over time – the portfolio appears to be safe. Needless to say, clear communications based on quantifiable portfolio risk metrics also serves as a check to verify that the asset allocation continues to conform to the investor's preferences and risk constraints.

Unconditional bankruptcy and conditional bankruptcy at a given time are just two of many potential risk measures that may concern an investor. Unconditional bankruptcy is not conditioned on time, so it disregards when the portfolio is depleted. However, the range and median time of bankruptcy[41] also provide valuable information about portfolio bankruptcy risk. If investors have assets outside their portfolio, they may not be overly concerned with bankruptcy as long as its duration is sufficiently short. Some investors panic if their portfolio value drops below a threshold value. Risk tolerance is investor specific, but knowing both the timing (late in retirement or early in retirement) as well as the magnitude of adverse events is critical. Knowing the range and probability of potential outcomes is a prerequisite for deciding if increasing investment risk is a prudent way to enhance the likelihood of a successful retirement.

Graph Three:

Graph Three incorporates all the assumptions made for graphs one and two except that the risk model generates initial portfolio returns in a bear market regime.



The portfolio is depleted in approximately 18 years, when she will be age 88, at the 50th percentile of start-in-bear-market outcomes unless, of course, the investor makes mid-course corrections. In this case, she will have approximately 6 to 7 years to deliberate on asset management strategy before the wealth-to-annuity-cost ratio declines to 1x coverage. If a stay-the-course "autopilot" strategy is maintained, she can expect to be

40 The risk modelling system also calculates the unconditional risk of bankruptcy which is defined as: (the number of trials manifesting surviving investors without funds) ÷ (the initial number of trials). Unconditional bankruptcy answers the question: how likely is it that an investor's portfolio will reach a zero value during his or her lifetime; conditional bankruptcy answers the question: how likely is it that an investor will be alive-but-broke at age x. In the instant case, the unconditional bankruptcy risk is approximately five percent. This suggests, given an original population of 70-year-old female investors taking an inflation adjusted monthly distribution of $5,150, a 95% chance of achieving the target lifetime income goal by adhering to the initial asset allocation / withdrawal policy guidelines.

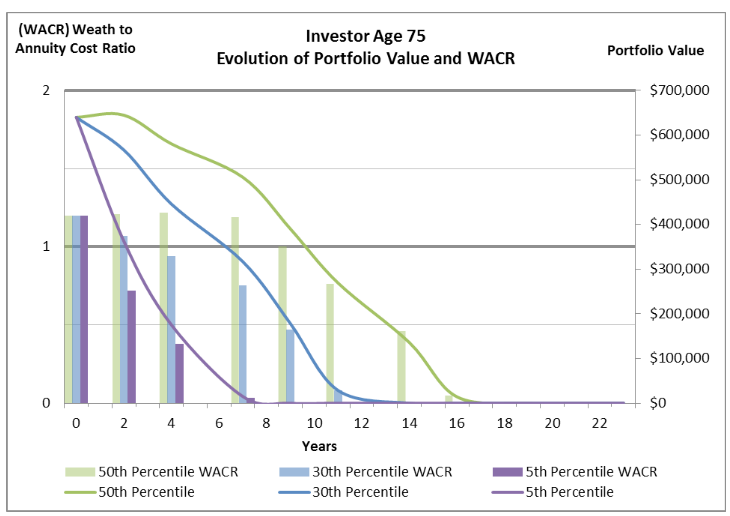
41 The risk modelling system provides insight into the average time alive-and broke, the standard deviation of the average time alive-and-broke, and the range of time alive-and-broke.

bankrupt by year 18 assuming that she remains alive. The 30th percentile and 5th percentile results are particularly sobering. In the worst (5th percentile) case, the investor is under 1x coverage by the end of the first year. She has lost flexibility over planning for her bequest and must immediately make some difficult decisions. In the median case, assets decline steadily over time – the inheritance is no longer being preserved, and asset management has become a race against time coupled with a growing possibility of financial catastrophe. A downward adjustment in spending is a particularly bitter pill because it fails to preserve cash flow required to support the minimum standard of living threshold. In this case, the investor may be counting on mean reversion of interest rates to preserve an annuity purchase option.

Given the historically poor predictive capability of equity market valuation measures such as price/earnings, dividend yield, and so forth, the planning implication is clear. The investor cannot confidently predict whether the next time period will be a bull or a bear market. The difficulty of making an accurate market forecast suggests that the prudent investor is increasingly conservative with respect to risky asset exposures; and, that the degree of conservatism is related directly to the level of wealth relative to consumption targets.[42] Although risky assets offer the temptation of high expected return, a decision regarding how much to allocate to risky assets should be made in a context that considers both the investor's level of dollar wealth and the value of the WACR. Making or maintaining the asset allocation decision based solely on a consideration of either age (glide-path rules) or shortfall risk metrics is suboptimal. Shortfall risk metrics reflect probabilities (hope for favorable future market conditions) rather than an assessment of the current portfolio's solvency profile. All else equal, the higher the WACR value, and the lower the current interest rate environment, the more appropriate is a tilt towards risky asset portfolios. A portfolio on the wrong side of the feasibility boundary, even if it has a high expected return and a substantial current dollar value, suggests that the investor may be unable to achieve long-term consumption and bequest objectives absent fortunate interest rate changes and capital market responses thereto. If the portfolio's value continues to deteriorate, the investment plan, which initially appeared to be prudent, becomes increasingly risky without providing the expectation of a reward to descendants. The key is vigilance; and the key to vigilance is an early understanding of when a seemingly healthy portfolio may, in fact, be on thin ice.

Graph Four:

We continue the bear market case study by supposing that five years have passed and that the investor's portfolio value is now $640,000, as depicted at the 50th percentile outcome in Graph Three. The investor faces a new set of circumstances: she is older, and her wealth is considerably diminished. Graph Four below incorporates this information and depicts potential financial outcomes. The analysis indicates that her portfolio is expected to last another 16 years, until she is 91.[43] Furthermore, she will be able to purchase an annuity anytime within the next 8 years, because the 50th percentile WACR value remains greater than 1. However, at the 5th percentile of outcomes, the task of funding a threshold living standard fully depletes the portfolio in roughly 7 years; and she is only one year away from technical insolvency and, hence, from violating the retirement income feasibility condition. If she is fearful of such an outcome, she can decide to exercise the option to annuitize provided that the risk model's pro forma yield curve is a reasonable benchmark for the actual interest rate term structure that the investor will face at her age 75. Not willing to, or perhaps unable to, sacrifice either her bequest goals or her chance of improving her budget constraint



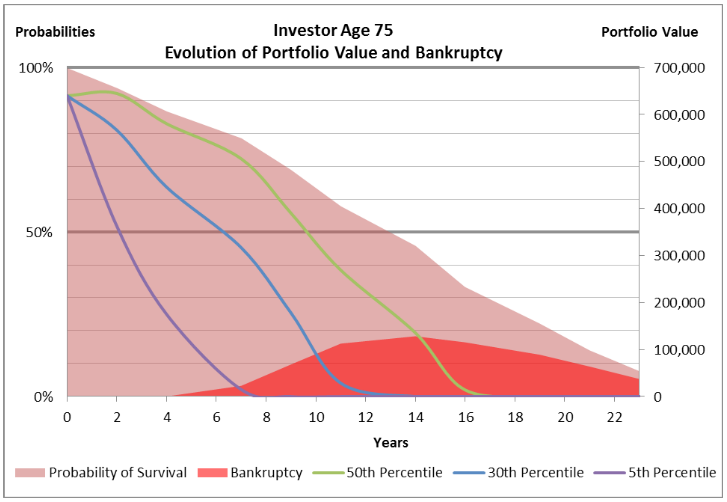
42 In a trust context, conservatism reflects a trustee's duty to manage assets with care, skill and caution.

43 The risk model assumes a random bull-bear selection for the sequence of returns starting at age 75.

by participating in a market rebound, she may forgo annuitization and continue to invest in the capital markets.[44]

Graph Five:

Graph Five provides insight into shortfall ("risk of ruin") probabilities.



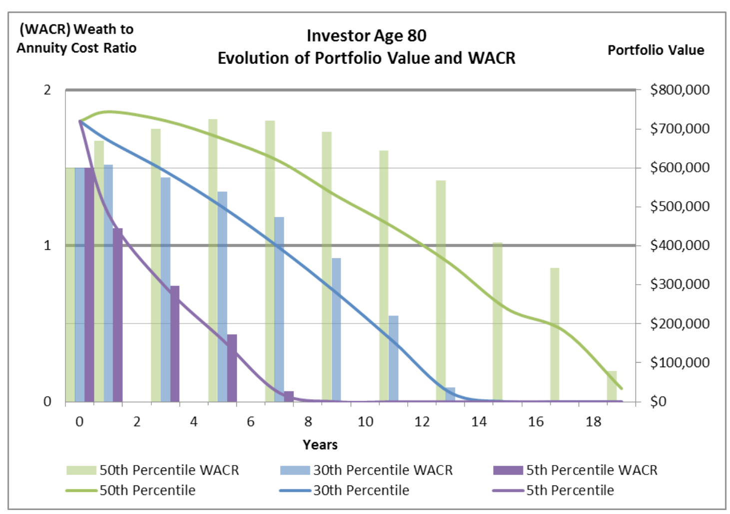
The likelihood of conditional bankruptcy given survival to age 85 is now nearly 15% (10% bankruptcy ÷ 69% survival). After twenty years it is approximately 57% (13% bankruptcy ÷ 22% survival). As age increases in this case study, the impact of market declines becomes more pronounced as consumption, geared to deplete principal as the Wealth/Consumption ratio becomes less favorable, takes an even bigger bite. As the Wealth to Consumption ratio value declines, the impact of a bear market becomes increasingly onerous. Avoiding a sequence of negative returns at the start of retirement does not mean that the investor is home free. Whenever the portfolio allows for the possibility of significant deterioration in the Wealth to Consumption ratio value, a bear market can prove financially devastating. Stated otherwise, sequence risk for modest-sized portfolios is present throughout retirement—not just at the beginning.

The bear-market case study depicts a race between principal depletion and the force of mortality. For modest sized portfolios, an older aged client may experience more volatility in 'sufficiency space' because they consume more principal whenever return expectations are not met.

Furthermore, tracking portfolio feasibility in terms of the WACR means that two identical portfolios can have very different risk characteristics because the distance to the free boundary—a 1x coverage ratio value—differs greatly as a function of age. The annuity cost for a 75 year old may be significantly less expensive than for a 65 year old. Investment risk is age related, as conventional wisdom suggests, but primarily as a function of the WACR value, not as a byproduct of a rule of thumb such as equity weighting should be approximately 100 minus current age.

Graph Six:

Suppose another 5 years pass and markets generate returns exceeding expectations. The investor is now 80 and her constant-dollar portfolio value is $720,000. Graph Six reflects the updated information. At the 50th percentile, her portfolio provides sufficient cash flow to age 100. However, at the 5th percentile, she runs out of money in 8 years. More importantly, at the 5th percentile, her WACR ratio drops below 1 in roughly two years. If the age-80 yield curve exhibits a two-year interest rate of 2% and a ten-year rate of 5%, she could purchase an annuity paying an inflation-adjusted $5,378 per month. Having been technically insolvent when she was 75, she may now decide that the potential for making a large bequest is not worth the short term stress of living with portfolio fluctuation, and she chooses to annuitize.[45]



44 We assume that she does not adjust her retirement income strategy. In reality, investors may change their portfolio allocation or modify their withdrawal distribution rate. A portfolio monitoring and surveillance system facilitates a prudent assessment of these asset management elections.

45 The annuitization decision may also reflect a variety of other factors including a concern with a continued ability to make complex financial decisions; an assessment that her heirs are financially secure absent further gifts or bequests; and other changes in personal circumstances.

The sample case study illustrates how a comprehensive retirement income portfolio monitoring and surveillance system facilitates prudent management of financial assets. The example tracks the risks metrics and decisions available during a period of market turbulence at the investor's ages 70, 75, and 80. Realistically, however, risk modeling should occur more frequently than five-year intervals.

VI. Asset Management Elections

A comprehensive surveillance and monitoring system uses a variety of risk metrics. These include performance evaluation metrics to evaluate investment strategies, shortfall risk metrics to ascertain the likelihood of success, and solvency metrics to assess portfolio sufficiency in terms of the feasibility condition. A comprehensive surveillance and monitoring system helps to frame asset management choices. In our case study, the bear market—combined with investment costs, fees and previous withdrawals—has pushed the portfolio's value from a high-water mark of $1.25 million to its current $1 million value. Despite the fact that the force of mortality now operates for an investor aged 70 rather than 68, the long-term shortfall probability risk has increased from 14% to 22%.[46] If bear market returns persist, the feasibility of critical financial objectives may no longer remain viable. However, if the market recovers recent losses, the free boundary will recede and the crisis may subside. There is no way to know which path will unfold.

The present-value cost of funding the threshold income is the monetary value of the free boundary. Does the investor purchase an annuity and invest the surplus in a growth-oriented portfolio? This solution is a variation on the classic CAPM two-fund solution first articulated as the "Separation Theorem" by James Tobin.[47] Should the investor segregate assets and create a two-fund portfolio—an annuity dedicated to providing minimum cash flow, and a portfolio of higher risk assets dedicated increasing wealth?[48]

The actuarial argument is that a single premium immediate annuity places the current beneficiary on a secure consumption path given that the portfolio owns a limited amount of investment capital.[49] An annuity allows the investor to capture the mortality risk premium (the extra periodic payment available as deceased annuitants default their periodic payouts to the remaining living annuitants). The annuity contract, assuming reasonable loads, permits the older investor to purchase a lifetime income relatively cheaply—annuities can maximize the cash flows available from a limited amount of investment principal. This leaves more funds available to invest in a growth-oriented portfolio of risky assets. However, from another perspective, an annuity is the ultimate "autopilot" withdrawal formula which permanently locks in the budget constraint, and which impairs the investor's ability to control his or her wealth.

Investing generally involves an exchange of one type of risk for another. With an annuity strategy, the investor agrees to capital sacrifice in exchange for lifetime income. Just as an investor might hedge downside market risk through acquisition of a derivative contract (short futures, long puts, etc.), so also, acquisition of an annuity contract hedges systematic longevity risk. In exchange for this benefit, however, the investor abandons the potential for future investment gains and enters into an illiquid and irrevocable financial arrangement with an insurer. The decision to pursue an actuarial solution—even within the structure of a two-fund solution of a risk-free annuity portfolio and a growth-oriented portfolio—should not be made cavalierly. Trading current wealth for an equivalent lifetime income stream may make the investor vulnerable

46 Shortfall risk from a trustee's perspective is discussed in Fast, Steven M., Gianopulos, Christiana N., & Macauley, Leiha, "Prudence—From Fuzzy to Precise" ALI-ABA Course of Study: Representing Estate and Trust Beneficiaries and Fiduciaries, (2007), p. 171.

47 Tobin, James, "Liquidity preferences as behavior towards risk," The Review of Economic Studies, Vol. 67 (1958), pp. 65 – 86.

48 An annuity, in this context, approximates a risk-free asset. However, it generates a level of lifetime income (assuming reasonable fees and charges) greater than available through replicating a risk-free bond portfolio—e.g., a series of zero-coupon treasuries. The annuity, as a contractual obligation of an insurance carrier, however, is not risk free because there is a probability of default. State insurance guarantee funds may mitigate losses up to a specified amount, but such funds are not direct obligations of state governments.

49 An optimal consumption path is not necessarily a constant income stream. In many life-cycle models, utility is maximized by keeping the marginal utility of consumption, not the dollar value of expenditures on goods and services, steady. This essay defines target expenditures as those required to maintain a minimum acceptable standard of living.

to unforeseen contingencies should he or she encounter medical emergencies or other needs not met by regular payments over a lifetime. The exchange of risks must be prudently evaluated.[50]

A bear market forces owners of modest-sized portfolios to consider a variety of planning options, none of which are ideal:

1. The investor can stay-the-course and continue to invest according to the risk/reward guidelines established by the initial investment policy. The danger in such a course is that, in the words of the CFA Institute, the investor may suffer "…an absolute level of loss that would completely derail an investment portfolio…." If the dollar value of portfolio assets penetrates the free boundary, they can hope for a market recovery of sufficient magnitude to restore the portfolio's long-term viability. However, there is no guarantee of such a result.

2. The investor may conclude that a portfolio with a low Wealth-to-Consumption Ratio requires higher expected returns, even at the risk of speeding up the time to portfolio depletion. The investor will elect to increase exposure to growth-oriented assets in order to make up recent losses. This is especially helpful if the bear market terminates and there is a switch to a more favorable investment climate. However, increasing expected future returns comes at the cost of increasing portfolio volatility. If the favorable conditions fail to materialize, shortfall probabilities may increase dramatically.

3. The portfolio owner can pursue a two-fund solution. This might take the form of moving the bulk of portfolio assets to cash in order to mitigate further investment losses. However, the opportunity cost of remaining in cash during low-yield environments may be so high that such an attempt to preserve principal merely exacerbates longevity risk.[51]

4. If an effective monitoring and surveillance system is in place, the investor might elect to implement a dynamic asset allocation program where equity risk exposure is a function either of (a) a pre-set floor or, (b) of the distance from the free boundary's location. As a bear market unfolds, a dynamic system typically moves the portfolio toward cash. Such a system—also commonly referred to as Constant Proportion Portfolio Insurance—is not commonly employed by investors with modest wealth.[52] It is more common to implement buy and hold or constant asset mix portfolio management approaches. Modest-sized portfolios may incur far greater expenses under a dynamic allocation approach, with the extra costs perhaps vitiating the risk control advantages.[53]

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50 It is worth noting that the existence of an actuarial safety net—irrespective of whether an investor chooses to annuitize some or all of her wealth—changes the investment decision making process. This point is stressed by Gaobo Pang and Mark Warshawsky in their 2010 study: "Optimizing the Equity-Bond-Annuity Portfolio in Retirement: The Impact of Uncertain Health Expenses." Op. Cit. They point out that, were annuities not available, households would substantially reduce their exposure to equity. Thus the possibility of securing a floor income through an option to purchase an annuity contract makes a risk-averse household more comfortable with equity risk exposure irrespective of whether the option is ever exercised. Of course, the danger of not exercising the annuity option is that the portfolio's future value will be less than the future cost of securing the target-income annuity.

51 Menoncin, Francesco and Scaillet, Olivier, "Mortality risk and real optimal asset allocation for pension funds," FAME Research Paper Series from International Center for Financial Asset Management and Engineering (2003), reviews multiple fund solutions ranging from Tobin's two-fund solution and Merton's three-fund solution to their own model, which is a five-fund solution. There are many "risk-free" solutions other than cash. The investor may consider laddering CDs, implementing sequences of zero-coupon bonds, and other fixed-income strategies. We note that a precondition for implementing cash matching and immunization strategies is that portfolio's market value exceeds the present value of the liabilities to be defeased. This is, however, merely a restatement of the feasibility condition.

52 There are important differences between a CPPI asset management approach and an approach based on a distance-from-boundary risk metric. CPPI focuses on asset levels only; a distance-from-boundary approach considers the dynamic interplay between asset and liability values. CPPI sets a floor to protect wealth; distance-from-boundary approach identifies a boundary to protect income.

53 A more detailed discussion of dynamic asset allocation approaches for trust management is found at Collins, Patrick J. & Stampfli, Josh, "Managing Private Wealth: Matching Investment Policy to Investor Risk Preferences," The Banking Law Journal vol. 126 no. 10 (November/December 2009), pp. 923 – 958. Jason S. Scott and John G. Watson: "The Floor-Leverage Rule for Retirement," Financial Analysts Journal September/October, (2013), pp. 45-60 suggest establishing a fixed-income investment floor and investing excess wealth using a 3x leveraged equity ETF.

5. Another form of a two-fund solution is a division of the investment corpus into an annuity to provide secure lifetime income and a performance-oriented portfolio to provide growth opportunities. This is a buy-an-annuity / invest-the-difference approach, which parallels a buy-term-insurance / invest-the-difference approach to asset accumulation.[54]

6. The worst of all possible worlds is to discover that the free boundary has been breached—i.e., the current market value of assets is less than the stochastic present value of liabilities. Use of the annuity cost metric for the free boundary makes sense when there are surplus funds. However, in the case of insufficient funds, asset management becomes more of a gambling question, i.e., does the investor want to accept a certain but unhappy outcome or risk a worse disaster with a more risky asset portfolio? The untenable nature of this situation highlights the importance of portfolio monitoring and surveillance policies.

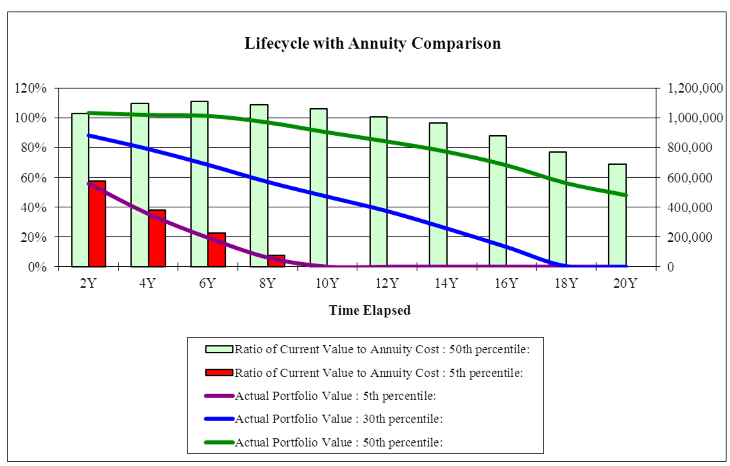
Developing prudent responses in the face of unattractive alternatives requires clear and accurate information. A disclosure of the nature and magnitude of the hurdles currently facing the portfolio, and an intelligent discussion of planning alternatives, can help investors decide how best to proceed.

VII. The Remainder Interest and the Steady State Ratio

This section explores the sometimes complex relationships between age, annuity costs, and the feasibility condition. The question of interest is this: what WACR is required assuming a two-fold goal of lifetime income plus preservation of principal on an inflation-adjusted basis? This issue is important for (1) investors assigning equal utility value for both lifetime income and terminal wealth; and, (2) trustees tasked with the duty of impartiality between beneficiary classes. We consider two investors, a female age 50 and a female age 70, each with a starting coverage WACR of around 1x. Each could convert immediately to an annuity and leave no inheritance, or each could hold the investment portfolio and try to match their periodic consumption to the equivalent annuity payments. Assuming equal starting portfolio dollar values, the periodic annuity benefit payable to the age 70 investor is substantially higher than that payable to the 50 year old. This is equivalent to saying that there is a greater mortality premium available at the older age.

The first graph depicts the evolution of wealth for the 50 year old assuming an initial portfolio value of $1 million, and a constant dollar $5,000 per month withdrawal. The coverage ratio (ratio of current portfolio value to the cost of an annuity) assumes that the equivalent annuity monthly payment ($5,000) grows by 3% per year. Although there is a 'basis' discrepancy between portfolio withdrawals adjusted for actual inflation and the 3% graded annuity payout, we do not assert that the example presents a precise apples-to-apples comparison. Rather, we are interested in obtaining insight into the feasibility condition at various ages and for various financial objectives.

Graph One:

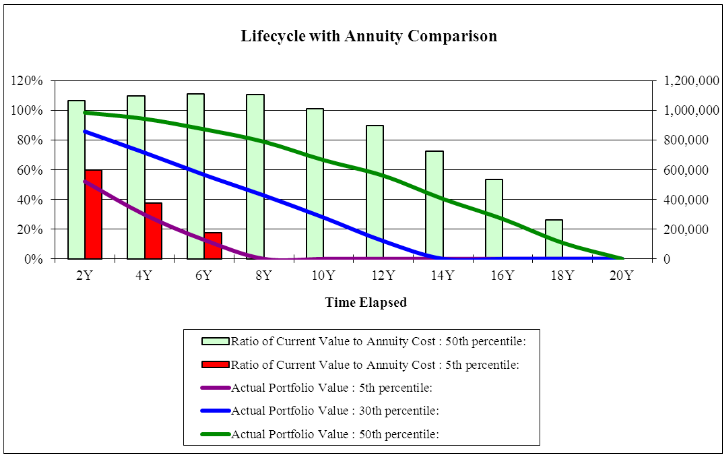


The columns depict the ratio of portfolio value to the cost of an annuity at various percentiles of the distribution of constant-dollar portfolio value net after periodic withdrawals. The right-side axis measures net portfolio

54 There is strong evidence suggesting that, for many investors, an annuity option is not attractive despite its possible economic advantages. A 2012 study by Suzanne Shu, Robert Zeithammer, and John Payne ["Consumer Preferences for Annuities: Beyond NPV"] found that of the 363 survey participants, 22% did not choose any annuity option despite the fact that all annuity options offered an expected payout with an NPV of at least $160,000 and with some options offering an NPV over $200,000 for a $100,000 purchase. "This strong dislike of annuities with a high benefit relative to upfront costs (more than would ever be offered in the market, in fact) suggests some individuals are unwilling to consider annuities regardless of the benefit offered." Paper available at www.anderson.ucla.edu/faculty/suzanne.shu/Shu_Zeithammer_Payne_annuity_preferences.pdf

value evolutions which the graph illustrates with solid lines.

The second graph assumes a 70 year old female who, from an initial portfolio valued at $1 million, withdraws an inflation-adjusted $6,500 per month increasing at a 3% annual rate.[55]

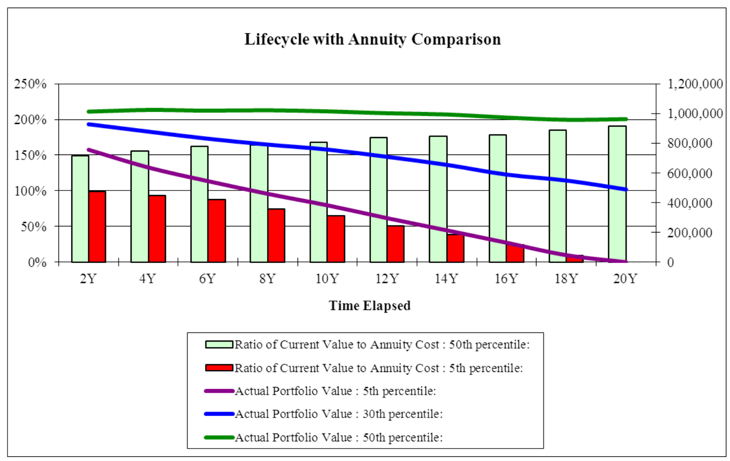


Both graphs illustrate the risk of trying to maintain a constant dollar lifetime income and an inheritance. At the assumed withdrawal rates, neither investor can expect to match the annuity payout AND protect the inflation-adjusted portfolio value for descendants. In other words, an investor incurs an expected cost for the option to provide a bequest. The older investor's wealth decays much faster – in the median case she is bankrupt at 20 years. This is not surprising because, when matching the annuity's periodic payout, the 'bequest option' is cheaper for younger people.[56]

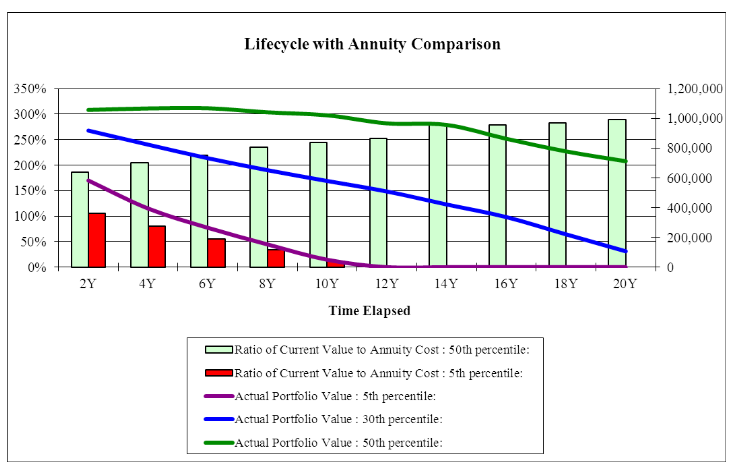
But this raises a related issue: what is the "steady state" WACR for different ages—i.e. what wealth-to-annuity coverage ratio needs to be maintained, in the median case, to generate the expectation of both a lifetime income and an inheritance? This is a critical issue for trustees trying to produce income and, simultaneously, to preserve the value of the trust corpus. A trustee would benefit greatly from understanding the feasibility condition—i.e., what wealth to annuity coverage ratio is required lest the trustee attempt to satisfy the legitimate expectations of the Settlor and beneficiary classes when, financially, the trust is technically insolvent.[57] It turns out that the answer depends on age – the older the investor, the higher the required steady-state coverage ratio. Although a potential strategy for safe retirement income is to adjust consumption to always be equal to the age-dependent steady state WACR, this is an extremely conservative strategy in that the goal is to sustain consumption rate in the median case forever.

To what degree must an investor or trustee adjust periodic income to lock in both a constant-dollar-income, and a preserve-the-value-of-the-corpus expectation? The following graphs approximate age-dependent, steady state coverage rates (Graph Three for a 50 year old; Graph Four for a 70 year old) for a $1M portfolio.

Graph Three:



Graph Four:



55 The cost for $5,000 increasing yearly by a factor of 3% is $1 million for a 50-year-old female annuitant. A 70-year-old female annuitant can purchase $6,500 for the same premium amount.

56 Refer to discussion of Graph Four which illustrates the impact of age on the evolution of the WACR.

57 The trust's distribution to the current beneficiary may, for example, be the amount deemed necessary under an ascertainable standard provision (health, education, maintenance and support).

Given an initial portfolio value of $1 million, the 50 year old female investor requires an approximately 1.5x coverage ratio to preserve the corpus at a $3,500 monthly withdrawal rate (graded upwards by 3% per year). There is evidence of a modest decay in constant dollar portfolio value over the 20-year planning horizon. The risk model indicates that the wealth/annuity cost coverage ratio increases slowly from approximately 1.5 to 2x coverage at the end of 20 years. Graph three suggests that given a withdrawal rate target of 4.2% adjusted for inflation ($3,500 x 12 = $42,000 per year), the hypothetical 50 year old female investor can expect to maintain the inflation-adjusted value of the portfolio ($1 million) for 20 years.

Assume that she is now age 70 and has maintained the constant-dollar equivalent of a $1 million portfolio. At age 70, a $4,000 monthly withdrawal rate with an approximately 2x coverage ratio works for the forthcoming 20-year period. Despite the higher monthly income, the WACR is now between 1.5 and 2—largely due to increased mortality credits for older age annuitants. The higher coverage ratio suggests that the 70 year old is at a comfortable distance from the free boundary and, therefore, can finance more monthly consumption from her portfolio. But, there's an important difference—the 70 year old, despite starting the forthcoming 20-year planning horizon with a coverage ratio well above a value of one, can't maintain the upwardly revised 4.8% inflation-adjusted withdrawal rate ($4,000 x 12 = $48,000 per year) and, simultaneously, maintain the inflation-adjusted portfolio value. Bequest motives significantly change the location of the free boundary. Given portfolios of equivalent initial value, the initial steady state coverage ratio per dollar of cash flow increases as age increases. At 50, a 1.5x coverage ratio is enough, but by 70 it should be closer to 2.5x, while 80 may require a 3.5x or 4x coverage ratio. In other words, assuming the wish to maintain the portfolio's constant-dollar value, consumption needs to be even less than the *current* age steady state ratio might suggest because the ratio must grow for the bequest to be sustained.

VIII. Hybrid Withdrawal Strategies

The previous examples assume a fixed, constant dollar, periodic withdrawal. This section reviews results from more flexible spending policies. Rather than developing sets of rules that may be complex and difficult to implement, we reduce the concept of 'adaptive portfolio withdrawals' to a single hybrid distribution policy. Specifically, the issue of interest is the merits and shortcomings of combining a fixed, constant-dollar withdrawal program with a floating, percentage of corpus program.[58]

Specifically, we compare withdrawal strategies for a hypothetical 50 year old female with an initial portfolio value of $1 million dollars. For each withdrawal strategy we simulate a diversified portfolio where trial length is the date of death or 20 years, if longer. The first table presents end-of-trial portfolio values for selected percentiles within the distribution of results; the second table presents the evolution of periodic distributions. We consider five retirement income withdrawal strategies:

- 3% of initial value fixed plus 1% floating ($30,000 + 1%)
- 2% of initial value fixed plus 2% floating ($20,000 + 2%)
- 1% of initial value fixed plus 3% floating ($10,000 + 3%)
- 0% of initial value fixed plus 4% floating (4%)
- 0% of initial value fixed plus 6% floating (6%)

58 A percentage of corpus withdrawal strategy is also called a 'unitrust' distribution election in a trust and estate planning context.

Table One:
Bankruptcy Risk Metric / Terminal Value / Cumulative Income

	$30k + 1%	$20k + 2%	$10k + 3%	4%	6%
Bankruptcy	12%	7%	3%	0%	0%
Terminal Portfolio Value					
50th Percentile	$1,204,284	$1,133,414	$1,178,610	$1,169,873	$803,924
30th Percentile	$517,918	$539,706	$628,465	$664,052	$434,090
5th Percentile	$0	$0	$57,908	$133,801	$73,650
Cumulative Income					
50th Percentile	$811,667	$820,333	$835,596	$858,571	$1,079,074
30th Percentile	$738,360	$682,740	$649,501	$621,174	$800,868
5th Percentile	$391,280	$369,857	$349,034	$276,124	$379,437

Table Two:
Range of Periodic Inflation-Adjusted Income

Annual Distribution at Year:	4	8	12	16	20
50th Percentile Distribution					
30k + 1%	$40,550	$41,100	$41,431	$41,459	$41,897
20k + 2%	$40,990	$42,053	$42,845	$42,719	$42,622
10k + 3%	$41,450	$42,987	$44,008	$44,153	$43,859
4%	$41,802	$44,234	$44,464	$45,181	$45,069
6%	$59,950	$58,746	$54,634	$49,851	$46,302
5th Percentile Distribution					
30k + 1%	$35,470	$32,943	$31,340	$23,020	$12,079
20k + 2%	$30,658	$26,326	$23,236	$21,084	$14,498
10k + 3%	$26,399	$19,437	$16,178	$13,706	$11,970
4%	$21,920	$13,278	$8,962	$6,622	$5,240
6%	$31,125	$16,256	$10,317	$6,204	$4,251

The fixed portion of the withdrawal strategy is inflation adjusted. The percent of corpus portion, although also expressed in constant dollar terms, is a percentage of the rolling 36 month portfolio value average.[59] Averaging portfolio values over a rolling 36-month window mitigates the effects of sudden large market swings on any single period withdrawal. In fact, the length of the smoothing function depends highly on the investor's preference for budgetary certainty—the acceptable level of variance in periodic withdrawals. At one extreme, an investor who does not value budgetary certainty may select a spending strategy reflecting a percentage of the portfolio's value at the time of withdrawal. On the other hand, an investor who values budgetary certainty may elect to employ a smoothing function with a longer length. An investor unwilling to cope with any variability in withdrawals avoids a floating strategy and elects a fixed, inflation-adjusted, withdrawal plan.[60]

Higher amounts of fixed withdrawals yield smaller ranges of periodic income, especially in earlier years. The tradeoffs for budgetary certainty, however, are higher bankruptcy rates and lower terminal portfolio values in worse case scenarios. These tradeoffs occur because percent of corpus strategies feed the proverbial 'bear' less—smaller amounts of cash are withdrawn from the portfolio as its value drops in a bear market. Withdrawal policies employing only a percent of corpus strategy—i.e. the 4% and 6% polices—will, ignoring any fixed portfolio management costs, have no chance of bankruptcy. The impossibility of bankruptcy may be a misleading indicator of risk, however, if the investor requires the portfolio to provide a threshold income level.

Taking less from the portfolio in bad years also allows it to recover faster and thus grow more in subsequent good years. This is manifest in the growing periodic withdrawals at the 50th percentile. The periodic growth leads to higher cumulative consumption for percent of corpus distribution policies at the 50th percentile. In average economic outcomes, a percent of corpus distribution policy, given a sufficiently low withdrawal rate, is akin to a "back loaded" retirement income plan—where the investor plans to

59 Prior to the 36th month, the distribution is calculated as a percentage of portfolio value averaged over the number of elapsed months in the sample trial beginning with month 1 and ending with month 35. The use of a 36 month smoothing function is for expositional purposes; we do not claim that it is the "best" smoothing length.

60 Under this fact pattern, a trustee charged with the duty to balance the interests of current and remainder beneficiaries may elect a 5% unitrust withdrawal formula for a total return trust. The expected value of the inflation-adjusted aggregate withdrawals is close to the expected value of the inflation-adjusted remainder interest over the applicable planning horizon. This type of analysis is a good way to document the impartiality of trust administration.

spend more cash later in life. If, however, the percent of withdrawal exceeds the portfolio's expected growth, the strategy becomes a "front loaded" retirement plan—where the investor plans to spend less cash later in life.

The ability to model hybrid withdrawal elections is an important pre-requisite to matching portfolio administration with investor return and risk preferences. Historically, retirement income modelling assumed that the investor exhibits only constant relative risk aversion. However, discussions concerning strategies to enhance portfolio sustainability differ from discussions concerning how to optimize utility of consumption for a retired investor with finite resources. Portfolio sustainability, when defined as the ability to fund a minimum periodic target income, implies a state preference utility function. That is to say, a retired investor may have a strong preference for avoiding periods during which consumption falls below a minimum acceptable threshold. Such an investor is willing to sacrifice greater utility in higher-portfolio-value states in favor of assuring a minimum standard of living in lower-portfolio-value states. Aggregate utility—summed over all consumption/investment states—takes a back seat to assuring, at least, a minimum living standard in each state. Optimization of expected utility, in most retirement income risk models, is a probability-weighted value taken over the entire distribution of outcomes—i.e., over all possible economic states from depression to prosperity. Conclusions derived from optimization procedures may differ drastically from those drawn from sustainability analysis. For example, an optimal withdrawal rate in a study by Finke, Pfau, and Williams requires that the utility-maximizing investor accept only a 43% sustainability rate.[61] Typically, commentators tracking shortfall risk metrics would consider such an optimal withdrawal strategy to be unacceptable.

Front- and back-loaded retirement spending plans also assume investor utility functions that differ significantly from utility functions conformable to constant relative risk aversion. Adjustments in retirement spending may, for example, reflect a decreasing risk aversion function,[62] or a high subjective time preference rate ("Fisher utility") which dictates a preference to enjoy life today rather than pile up money for a low survival-probability future.[63] A flexible retirement income model enables the planner to elicit an investor's wealth and spending preferences not in terms of a mathematical expression for a utility of spending and bequest function, but rather in terms of an implied utility function reflecting a preferred and coherent retirement portfolio strategy.

IX. Conclusion

Monitoring and managing a retirement income portfolio presents complex challenges. This paper demonstrates why performance monitoring and reporting should provide both forward looking risk metrics—shortfall probability risk—and current risk metrics—portfolio solvency risk. Although of some interest to retired investors, tracking performance efficiency via Sharpe and Information Ratios is of less value than monitoring and reporting whether the portfolio is likely to run out money. Likewise, reporting in benchmark-relative terms (the portfolio beat the S&P 500) is of less importance than reporting in terms of the continued feasibility of client goals. To date, there is a small library of books and articles on asset allocation strategies and withdrawal formulae for retirement income portfolios. However, there is only a scant amount of advice on how to monitor wealth to assess whether goals continue to remain attainable. One can argue, however, that this should be the primary focus of retirement income portfolio management. If current portfolio value is less than retirement liabilities, the portfolio is technically insolvent. Investors can hope that things will work out satisfactorily, buy they cannot expect them to do so.

The paper advances the proposition that retirement income portfolio monitoring can be greatly enhanced by basing decisions on *current observables* rather than on

61 Finke, M., Pfau, W. D. and Williams, D., "Spending Flexibility and Safe Withdrawal Rates," Journal of Financial Planning, vol. 25, no. 3 (2012, March). pp. 44 - 51. Similar observations are found in Tomlinson, Joseph A., "A Utility-Based Approach to Evaluating Investment Strategies," Journal of Financial Planning (2012) available at www.fpanet.org/journal/AUtilityBasedApproach/

62 See, for example, Venter, Gary G., "Utility with Decreasing Risk Aversion," (1983).

63 Milevsky, Moshe and Huang, H., "Spending Retirement on Planet Vulcan: The Impact of Longevity Risk Aversion on Optimal Withdrawal Rates," Financial Analysts Journal vol. 67, no. 2 (March/April, 2011), pp. 45 – 58.

past results or future forecasts. If assets are greater than liabilities, economic objectives remain feasible because the portfolio has surplus wealth. Asset value is the current portfolio market value. The value of liabilities is the present value cost of providing a safe, sustainable, and sufficient income and, depending on the investor's goals, providing a target bequest. But this determination is an actuarial calculation as well as an investment projection. Therefore, the portfolio is best managed not to a rate-of-return bogey, but to an asset/liability management objective subject to a constraint that surplus should not turn negative.

The most accurate and appropriate liability benchmark is the current cost of lifetime income as provided by an annuity contract. Annuity cost is a current market observable and does not depend on the accuracy of market forecasts, projections of the future expected equity and bond market risk premia, or the credibility of retirement income risk models. There is no guesswork about the economic condition of the portfolio—if the portfolio violates the feasibility condition both the retiree and her heirs face a bleak economic prospect. In these circumstances, it is cold comfort to discover that the portfolio's rate of return beat a benchmark return series. We use the term 'Free Boundary' to define the point at which the market value of wealth exactly equals the present value of liabilities. If an investor does not know the location of the Free Boundary, portfolio administration is based largely on hope rather than on a solid financial assessment.

Finally, we provide some case studies. Neither current asset values nor the investor's return forecasts are of primary importance in monitoring the economic viability of the retirement income portfolio. Rather, the ratio of current wealth to annuity cost is the key risk metric. We term this risk metric the 'Wealth to Annuity Cost Ratio' [WACR]. The value of the WACR risk metric changes over time; and we illustrate a dynamic method for integrating solvency monitoring with retirement income risk model outputs.

We conclude that prudent portfolio management benefits from knowing two sets of information: (1) the likely range of future wealth given periodic withdrawal demands—a sustainability test; and (2) the distance of current wealth from the Free Boundary that separates investor objectives into regions of feasibility and infeasibility—a solvency test.

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Appendix: Pricing Annuities in the Retirement Income Risk Model

Pricing annuities in a risk modelling system presents significant challenges. The risk model described in this paper uses a pro-forma yield curve based on four parameters—the current level of inflation, the real short term rate premium to inflation, the real long term rate premium, and the curvature from short to long term rates. A fundamental assumption is that both of these premiums (short term to inflation and long term to short term) remain constant over time. The pro-forma yield curve is not the current term structure of interest rates. Rather it produces an "expectation"—based on current inflation and historical risk premia—of what an investor might reasonably expect to pay, on average, for an annuity contract.

In the case of annuities, this assumption may actually be superior to assuming time varying premiums reflecting forecasted changes in the yield curve's interest rates and shape. The purpose of the WACR calculation is to illustrate anticipated coverage ratios of portfolio assets to annuity costs. Introducing additional variance to the pro-forma yield curve model would allow for a range of possible premiums which, in turn, would create noise around the expected annuity cost, and therefore noise around the expected portfolio / annuity cost ratio [WACR]. Therefore, the absence of variance is likely to provide a clearer answer to such questions as 'Will the investor be able to maintain a portfolio above the minimum asset level required to buy a replacement annuity' without biasing the answer either positively or negatively.

The rate model used to price annuities is independent from the returns calculated for fixed income instruments. Both equity and fixed income returns are modeled on one month normally distributed (and appropriately correlated) returns within the appropriate economic regime—bull or bear. The annuities are priced on a pro-forma, non-varying forward yield curve model. As a result, the change in the price of an annuity is based on changes to the annuity buyer's age and on the current rate of inflation. During periods of high inflation, annuities are relatively cheaper as future payments are discounted at a higher rate. During periods of low inflation the converse is true. Of course, the monitoring system should periodically input updates to age and interest rates in order to recalculate annuity costs and, by extension, current and anticipated coverage ratios.

Modeling the evolution of the wealth-to-annuity-cost ratio requires several steps. The first step is to determine the 'Starting Benefit Payment / Starting Price.' The 'Starting Benefit Payment' component equals the starting monthly benefit amount for an immediate annuity purchased by the investor where the payment is measured in today's dollars. The starting periodic annuity payment may occur at month one (if the investor immediately opts to exchange financial wealth for annuity income) *or, if the investor decides to defer the annuity purchase, the payment may start at any month thereafter*. The "Starting Price' component automatically rolls forward month-by-month to project the annuity's cost as the investor ages—keeping current health status constant. Despite the nascent market for purchasing annuity and structured settlement income streams by private investment groups, we deem a decision to purchase an annuity to be an irrevocable wealth transfer.

The second modeling step is to 'Adjust Starting Benefit Payment.' If an annuity purchase decision is deferred, the investor is faced with the need to increase the future-date initial periodic payment to reflect a decrease in purchasing power. We adjust the future-date initial payment by the realized path of inflation as calculated by the retirement income risk model. For example, an investor with a $2 million current portfolio could elect to purchase the annuity today at a wealth/annuity income ratio of 2; or, she could wait and see how the inflation-adjusted wealth / adjusted annuity income coverage ratio plays out in the future. That is to say, the coverage ratio is dynamic. An investor with a current ratio above 1 can see the likelihood that the ratio might either improve or deteriorate over time given the current asset allocation, target withdrawal amount, and projected mortality credits embedded within the annuity pricing structure. If the 'Adjust Starting Benefit Payment' calculation is based on a risk model input of zero, then the model assumes that the annuity benefit is nominal rather than inflation adjusted. In a nutshell, the 'Adjust Starting Benefit Payment' calculation reflects the amount of the annuity payment *to be purchased*. This is a key input into calculating how much wealth must be exchanged—now, or in the future—for lifetime income. It is the information that the investor needs to consider in order to decide if and when annuitization is appropriate.

The final modeling step is to specify an input termed 'Growth after Start.' The 'Growth after Start' calculation reflects the amount the annuity income will change after the single-premium, immediate annuity's contract purchase date. Because the risk model treats future inflation as a stochastic variable, the model requires a reasonable input factor for increasing an annuity's future payouts once the annuity payout starts. If the input value is zero, the annuity payments, once started, do not increase. The risk model, in essence, asks the investor to elect an automatic yearly payout adjustment feature equal to 0%, 2%, 3%, 4% or some other reasonable increase factor. The larger the increase factor, the greater the cost of buying the annuity contract, all else equal. Although the specified rate of payment increase is unlikely to match precisely the realized inflation rate, it nevertheless mitigates adverse effects on future purchasing power.

Prior to periodic update reviews with clients, the application allows the advisor to input the actual current yield curve data, current investor age, annuity type (nominal or graded benefit), and other information in order to update the current cost of providing lifetime income via a commercial annuity contract. Thus, the application makes an important distinction between the pro-forma yield curve used for WACR projections and the actual yield curve data required for periodic portfolio monitoring and review. ▪